156

11

158

-: HAND WRITTEN NOTES:-

OF

ELECTRICAL ENGINEERING

1

-: SUBJECT:-

ELECTOMAGNETIC

THEORY

11

2

electromagnetics :-

electrostatics
--- static electrical fields
$\vec{E}, \vec{D} \neq f(t)$

Magnetostatics
--- static Magnetic fields
--- $\vec{B}, \vec{H} \neq f(t)$

time varying electrical & magnetic fields
$\vec{E}, \vec{D}$; $\vec{B}, \vec{H} \neq f(t)$

$\vec{E}$ -- static electrical field intensity
(N/C ; V/m)

$\vec{D}$ -- electrical field density
-- elec. displacement vector (C/m^2)

$\boxed{\vec{D} = \epsilon \vec{E}}$

permittivity of Meadium (F/M)

$\boxed{\epsilon = \epsilon_0 \epsilon_r}$

→ ϵ_0 --- permittivity of free space.

$\epsilon_0 = 8.854 \times 10^{-12}$ F/m
$= \frac{1}{36\pi} \times 10^{-9}$ F/m

$\boxed{\epsilon_r \geq 1}$

$\epsilon_r = 1$ -- for free space,
> 1 -- for any other dielectric

ϵ_r = Relative permittivity of meadium (unitless)
--- dieelectric constant

unless specified $\epsilon_r = 1$

$\vec{D}$ constant irrespective of type of Meadium.

Static Magnetic fields :-

$\vec{H}$ ---- Mag. field Intensity ---- (A/m)

$\vec{B}$ ---- Mag. flux density ---- ($wb/m^2 \equiv T$)

$$\boxed{\vec{B} = \mu \vec{H}}$$

μ ↑ permeability of meadium (H/m)

$$\boxed{\mu = \mu_0 \mu_r}$$

μ_0 = permeability of free space

$= 4\pi \times 10^{-7}$ --- H/m

μ_r = relative permeability of meadium

$$\boxed{\mu_r \lesseqgtr 1}$$ unit less

$\mu_r < 1$ --- diamag.

$= 1$ --- non-mag.

> 1 --- paramag.

$\left.\right\}$ $\mu_r \cong 1$ non ferromag.

$>> 1$ --- ferromag. $\left.\right\}$ $\mu_r >> 1$ ferromag.

unless specified

$$\boxed{\mu_r \cong 1}$$

{ Water (diamag.) ; $\mu_r = 0.999991$

Air (paramag.) ; $\mu_r = 1.0000006$

1) { cobalt (ferromag) ; $\mu_r \cong 250$

Fe (0.5% impure) ; $\mu_r \cong 5000$

Fe (0.05% impure) ; $\mu_r \cong 200000$

ferromag.

Time varying elect. & mag. fields :- (5)

$\vec{E}, \vec{D}$; $\vec{B}, \vec{H}$ = f(t)

Imp. points :-

(1) The maxwell's equations are a set of four eqns. Which a Relationship b/w time varying ele. & mag. fields

(2) When ever any wave propagate then the ele. field, mag. field & direction of propagation are mutually perpendicular each other.

$(\vec{E} \perp \vec{H}) \perp$ direction of propagation

⇓

TEM wave

Transverse em waves (uniform plane waves)

(3) When their is large mismatching b/w the length of filament & wavelength of operation at low freq. then entire power dissipation in element.

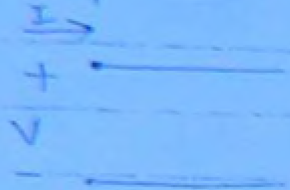
(4) at High freq the length is comparable to the wavelength of operation, then the power is radited through that element

(5) at low freqs the depth of penetration is High. & therefor we use thick Conductor, where as at High frequencys depth of penetration is low & therefor thin conductors are use

the power from the transmitter to the antina is transported with the help of co axial transmisson line or parallel wire transmisson line.

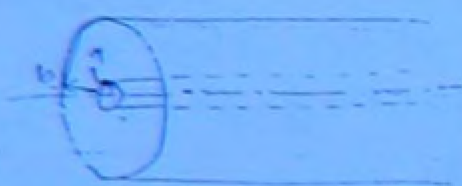
Transmisson line :-

1. parallel wire transmisson line



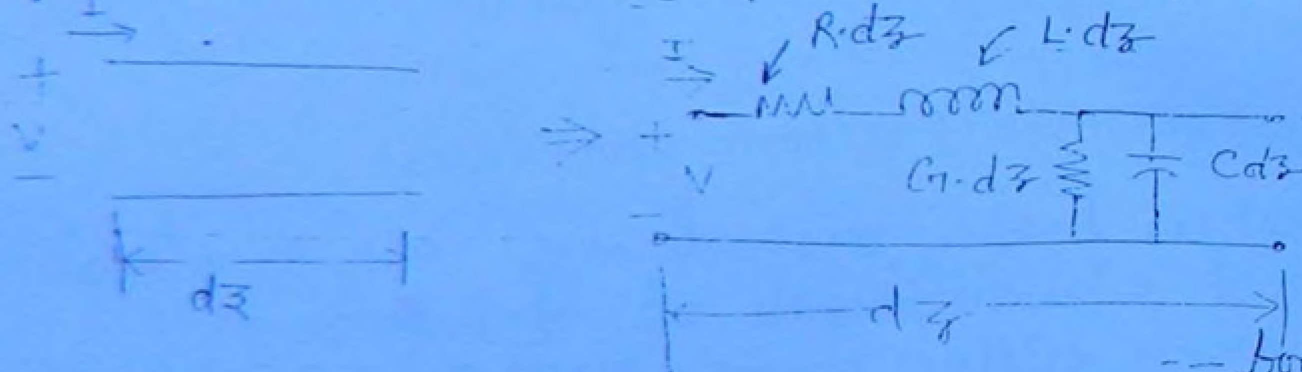
used as:
HT wires (High tension..)
telephone wires

Co-axial T-L



used in
CATV
as CRO load

Distributed parameters equivlent ckt. of T.L



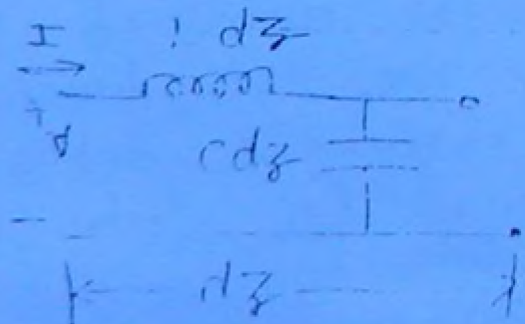
--- for lossy line

primary constant of the line

- R --- Ω/m
- L --- H/m
- C --- F/m
- G S/m

loss less line

$$\boxed{\begin{matrix} R = 0 \\ G = 0 \end{matrix}}$$



Imp. points :-

(1) Due to Inherint properties of lossy trans line it's assume that R, L, G & C are Effectively distributed along the enite length of T/L (7)

(2) Due to lossy nature of the line & due to finite Conductivity of the line

∴ Some losses occure on the T/L due to curre flow along the line.

the Resistor R is Responsible for total powe dissipation taking place due to lossy nature of the line

(3) due to current flow & due to mag. fields some Mag. energy will exists :-

the Inductor L is responsible for total Magnetic store In the TL

(4)

Due to potential differece b/w the two lines, some electrical fields & there for some ele. energy is finite in the TL. the capacitor C is responsible for the total electrical energy stor In the transmisson line.

(5) The meadium of dielectric b/w the two lines is in general it's lossy nature.

Some power dissipation take place as the current leaks through the lossy dirrective.

G is responsible for total power dissipiation taking place due to lossy nature of the dielectric b/w the TL

(6) for a lossless line R & G are zero.

(7) as the voltage or current wave form are interest on the TL the wave is attenuated exponentially and therefor the magnitude of the voltage & current will decrease as the wave propagate along the line.

8

Behaviour of V & I along the line:-

$$V = \underbrace{\underbrace{V^{+} e^{-\gamma z}}_{\text{propagating along } +z}}_{\text{Incident wave}} + \underbrace{\underbrace{V^{-} e^{+\gamma z}}_{\text{propagating along } -z}}_{\text{Reflected wave}}$$

V^{+} --- Amplitude of voltage wave propagating along $+z$ direction.

V^{-} --- Amplitude of wave propagating along $-z$

γ - propagation const. (Complex)

$\gamma = \alpha + j\beta$;

α - attenuation const. (nepers/m)

β - phase const (rad/m)

$$I = \frac{1}{Z_0}\Big(\underbrace{V^+ e^{-\gamma z}}_{\text{incident wave}} - \underbrace{V^- e^{+\gamma z}}_{\text{reflected wave}}\Big)$$

9

due to reverse direction of current

Z_0 --- Characteristic Impedance.

$\underbrace{Z_0 ; \gamma (= \alpha, \beta)}$ --- Secondary const of the line.

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \alpha + j\beta$$

Basic features: —

(1) as the voltage or the current waveform is impressed for a lossy line it's subjective to attenuation as well as phase change.

Therefor the magnitude of Voltage & current will decrease expoentially as the waves travell along the line.

if $\alpha = 0$ the wave propagation taks place without any attenuation.

therefor the Magnitude of Voltage & current will remain const. at all the points along the line.

(2) if $\beta = 0$ there is propagation & wave is entirly attenuate.

imp
3) The chst of the line represent the ratio b/w voltage & current at any point of any infinite long line (10)

4) The propagation const γ & the charts imp. is zero are represented by the secondary const of the line.

Since depend upon the primary const of R, L, C & G of the line.

Lossless line :-

$$R = 0$$
$$G = 0$$

$$Z_0 = \sqrt{\frac{\cancel{R}+j\omega L}{\cancel{G}+j\omega C}} = \sqrt{\frac{L}{C}} \quad \therefore \text{ real ; const } (\neq f(\omega))$$

$$\gamma = \sqrt{(\cancel{R}+j\omega L)(\cancel{G}+j\omega C)}$$

$$= j\omega\sqrt{LC} = \alpha + j\beta$$

$\alpha = 0$
$\beta = \omega\sqrt{LC}$
$\gamma = j\beta$

$$\beta = \omega\sqrt{L.C}$$
$$= \frac{\omega}{v_p} = \frac{2\pi}{\lambda}$$
$$v_p = \frac{1}{\sqrt{LC}} \quad \ldots \text{const.}$$

Summary :-

--- lossy line :

$$V = V^+ e^{-\gamma z} + V^- e^{+\gamma z}$$

$$I = \frac{1}{Z_0}\left(V^+ e^{-\gamma z} - V^- e^{+\gamma z}\right)$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

R, L, G, C ..... primary const

$\gamma (= \alpha, \beta)$; Z_0 --- sec. const.

---- lossless line

$$R = G = 0$$

$$\alpha = 0$$

$$\gamma = j\beta$$

$$\beta = \omega\sqrt{LC}$$

$$\beta = \frac{\omega}{v_p} = \frac{2\pi}{\lambda}$$

$$v_p = \frac{1}{\sqrt{LC}}$$ --- const.

$$Z_0 = \sqrt{\frac{L}{C}}$$ --- real ; const ; $\neq f(\omega)$

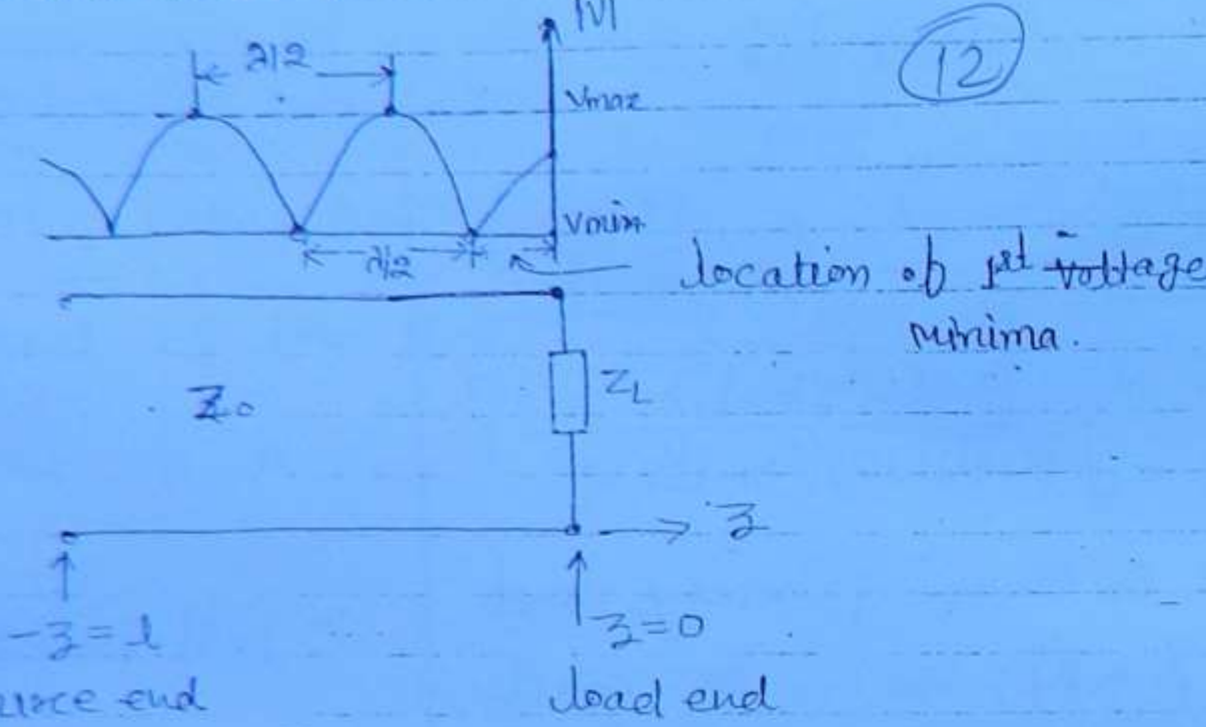
$$V = V^+ e^{-j\beta z} + V^- e^{+j\beta z}$$

$$I = \frac{1}{Z_0}\left(V^+ e^{-j\beta z} - V^- e^{+j\beta z}\right)$$

Imp

Terminated T.L.

(12)



Case 1 : $Z_L \neq Z_0$

--- mismatched line.

① Max. power is not transfer to the load.

② Incident & Reflected waves will exists.

③ standing wave patter will exists & there for Maxima & minima will exists along the line. there for there is a form of standing wave patteren.

④ coefficient of Reflection has a finite value.

⑤ due to V_{max} & V_{min} Voltage standing wave Ratio is finite.

$$\underbrace{VSWR}_{} = S = \frac{V_{max}}{V_{min}}$$

Voltage Standing wave Ratio

Case : 2 : $Z_L = Z_0$

---- Matched line.

(1) Maximum powr. is transferred from source to load.

(2) Reflection cofficient is zero.

(3) There is no reflected waves, no standing wave pattern, $V_{max} = V_{min}$ & there for the voltage along the line. is const. at all the points.

(4) The VSWR has a min. value of unity.

$$S = \frac{V_{max.}}{V_{min.}} = 1$$

---- S_{min}

since $V_{max.} = V_{min}$

To find :

(1) Reflection cofficient :- $\Gamma = \frac{V^-}{V^+}$ Complex

$$\underbrace{|\Gamma|}e^{j\theta} = \rho e^{j\theta}$$

(2) Transmisson cofficient :-

$$T = \frac{V_L}{V^+}$$

(3) $VSWR\,(S) = \frac{V_{max.}}{V_{min.}}$

(4) $Z_{in} = \frac{V_s}{I_s} = \left.\frac{V}{I}\right|_{-z=l}$

assume : line is lossless

(1) $V = V^+ e^{-j\beta z} + V^- e^{+j\beta z}$

$I = \frac{1}{Z_0}\left(V^+ e^{-j\beta z} - V^- e^{+j\beta z}\right)$

$z = 0$

$V_L = V^+ + V^-$ ——— (1)

$I_L = \frac{1}{Z_0}(V^+ - V^-)$ ——— (2)

$$\frac{V_L}{I_L} = Z_L = Z_0 \frac{V^+ + V^-}{V^+ - V^-}$$

$$\boxed{\Gamma = \frac{V^-}{V^+} = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

if $\frac{Z_L}{Z_0} = z_L$ .... normalized load impedance.

$$\Gamma = \frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1} = \frac{z_L - 1}{z_L + 1}$$

(5) Transmisson coefficient :-

$$T = V_L / V^+$$

from equation (1)

$$\frac{V_L}{V^+} = 1 + \frac{V^-}{V^+}$$

$$\boxed{T = 1 + \Gamma} = 1 + \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$T = \frac{2Z_L}{Z_L + Z_0} = \frac{2\bar{Z}_L}{\bar{Z}_L + 1}$$

(6) $S = \frac{V_{max}}{V_{min}} = \frac{|V^+| + |V^-|}{|V^+| - |V^-|}$

$$= \frac{1 + |V^-|/|V^+|}{1 - |V^-|/|V^+|} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$\boxed{S = \frac{1+\rho}{1-\rho}}$$

15

$\rho_{min} = 0 \; ; \; S_{min.} = 1$

$\rho_{max} = 1 \; ; \; S_{Max.} = \infty$

(4) $Z_{in.} = \left. \frac{V}{I} \right|_{-z=l}$

$$Z_{in} = Z_0 \left. \frac{V^+ e^{-j\beta z} + V^- e^{+j\beta z}}{V^+ e^{-j\beta z} - V^- e^{+j\beta z}} \right|_{-z=l}$$

$e^{\pm j\beta l} = \cos\beta l \pm j\sin\beta l$

$\frac{V^-}{V^+} = \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$

$$\boxed{Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}}$$

Summary :-

$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{\bar{Z}_L - 1}{\bar{Z}_L + 1}$

$Z_L = Z_0 \frac{1+\Gamma}{1-\Gamma}$

$T = 1+\Gamma = \frac{2Z_L}{Z_L + Z_0} = \frac{2\bar{Z}_L}{\bar{Z}_L + 1}$

$S = \frac{1+\rho}{1-\rho} \; ; \; \rho = \frac{S-1}{S+1}$

$0 \leq \rho \leq 1$; $Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}$

$1 \leq S \leq \infty$

Date

ex: A loss less line of length l & characteristic impedance is zero is terminated by a load impedance Z_L. calculate:

① Input impedance. (16)

② reflection coefficient.

③ VSWR when :-

$Z_L = 0$ ---- sc line

$Z_L = \infty$ ---- o.c. line

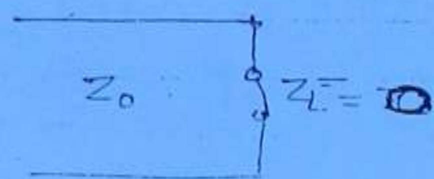
$Z_L = Z_0$ --- Matched line

$Z_L = jx$ --- purely reactive load.

Case : 1 $Z_L = 0$

--- sc line

$$Z_{in} = Z_0 \frac{\cancel{Z_L} + jZ_0 \tan\beta l}{Z_0 + j\cancel{Z_L}\tan\beta l}$$



$= jZ_0 \tan\beta l$

--- purely reactive

--- . S.C. stub line

$$Z_{in} = \boxed{Z_{sc} = jZ_0 \tan\beta l}$$

$$\Gamma = \frac{\cancel{Z_L} - Z_0}{\cancel{Z_L} + Z_0} = -1 = \frac{V^-}{V^+}$$

$\Gamma = \rho e^{j\theta} = -1$

$\rho = 1$

$\theta = \pi$

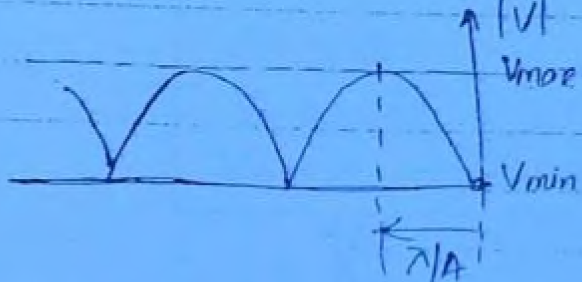
$\left\{ e^{j\theta} = e^{+j\pi} = \cos\pi + j\underbrace{\sin\pi}_{=0} \right.$

$= -1$

$$S = \frac{1+\rho}{1-\rho} = \infty = \frac{V_{max}}{V_{min}}$$

17

Minima is located at the load end.



S.C line :-

$Z_L = 0$

$\Gamma = -1$

$\theta = 180°$

$\rho = 1$

$S = \infty$

$Z_{sc} = j Z_0 \tan \beta l$

Features :-

1) The input imp. is purely is reactive in nature. for the shortest length of the line this imp. is inductive nature.
2) is stub line is a portion of line which has been S.C at the load end & has purely reactive o/p imp.
3) a Short circuit stub can be used for matching Transmission line with the load imp. for max. power transfer.
4) The reflected Voltage & incident Voltage are equal in Magnitude but are phase shifted by 180°
5) The Voltage minima occures at the load end & the 1st Voltage maxima occures at a distance of λ/4 from the load end.

Case : 2 $\quad \underline{Z_L = \infty}$

---- OC line

(18)

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}$$

$$= Z_0 \frac{1 + j(\cancel{Z_0/Z_L}) \tan\beta l}{(\cancel{Z_0/Z_L}) + j\tan\beta l}$$

$$\boxed{Z_{oc} = -jZ_0 \cot\beta l}$$

---- purely reactive.
--- OC stub line.

$$Z_{sc} = jZ_0 \tan\beta l$$
$$Z_{oc} = -jZ_0 \cot\beta l$$

$$Z_{sc} \cdot Z_{oc} = Z_0^2$$

$$\longrightarrow \boxed{Z_0 = \sqrt{Z_{sc} \cdot Z_{oc}}}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{1 - \cancel{Z_0/Z_L}}{1 + \cancel{Z_0/Z_L}} = 1 = \frac{V^-}{V^+}$$

$$\Gamma = \rho e^{j\theta} = 1$$
$$\rho = 1$$
$$\theta = 0$$
$$S = \frac{1+\rho}{1-\rho} = \infty = \frac{V_{max}}{V_{min}}$$

---- V_{max} occures at load end.

O.C line :-

$Z_L = \infty$

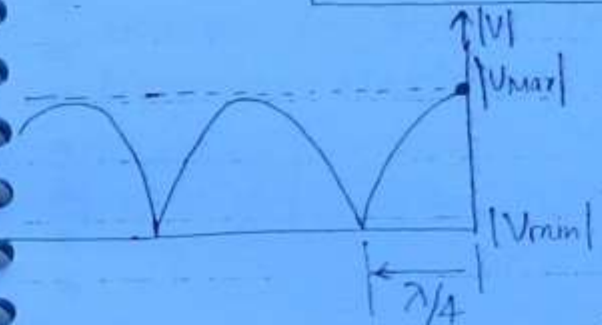
$\Gamma = 1$

$\rho = 1$

$\theta = 0^\circ$

$S = \infty$

$Z_{oc} = -jZ_0 \cot \beta l$



Features:

1) The input imp. is purely Reactive & for a shortest length of the line it's capacitive in nature.
2) The o.c. stub line can be used to Matched any transmission line with the load imp. for max. power transfer
3) The reflected & incident voltages has same Magnitude and are in-phase.
4) Voltage maxima occurs at the load end.
5) When the line is first s.c & then o.c. the voltage minima shifted by distance $\lambda/4$ from the load end towards a source end.
6) The charstic's. imp. of the line is a geometric Mean of input imp of the line when it's is s.c. & then o.c.
7) An Impedance inversion takes places when the line is 1st s.c. & then o.c & vice-versa.

Thetefor Inductive Imp. is transformed into capacitive inductive & vice-versa.

20

case : 3 : $Z_L = Z_0$

Matched line

$$Z_{in} = Z_0 \frac{\overset{=Z_0}{Z_L} + j Z_0 \tan \beta l}{Z_0 + j \underset{=Z_0}{Z_L} \tan \beta l}$$

$$\boxed{Z_{in} = Z_0}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = 0 = \frac{V^-}{V^+}$$

$$\Gamma = \rho e^{j\theta} = 0$$

$\rho = 0$

$\theta =$ Indeterminate $(0 \le \theta \le 360^\circ)$

$= 0^\circ$ (min)

$$S = \frac{1+\rho}{1-\rho} = 1 = \frac{V_{max}}{V_{min}}$$

$V_{max} = V_{min}$

Matched line :-

$$\boxed{\begin{array}{c} Z_L = Z_0 \\ Z_{in} = Z_0 \\ \Gamma = 0 \\ \rho = 0 \\ \theta = 0^\circ (min) \\ S = 1 \end{array}}$$

Feature:

① A perfectly matched line behaves as infinitely long line since in each case the i/p imp. of the line is equal to the chst. imp. of the line.

② There is no reflected waves, reflection coefficient is zero, VSWR has min. value of unity.

③ Maximum pow is transferred to the load so that Vmax = Vmin, there is no standing waves & therefore the voltage along the line is uniform at all the points.

$\boxed{\text{case: 4}}$ $Z_L = jx$

--- (purely reactive load)

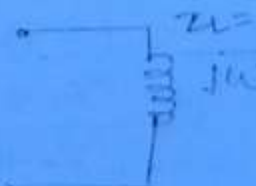
$$Z_{in} = Z_0 \frac{\overbrace{Z_L}^{\to jx} + jZ_0 \tan\beta l}{Z_0 + j\underbrace{Z_L}_{\to jx} \tan\beta l}$$

$$-x$$

$$Z_{in} = \underbrace{j\underbrace{\left[Z_0 \frac{x + Z_0 \tan\beta l}{Z_0 - x \tan\beta l}\right]}_{\text{Real}}}_{\text{pure reactance}}$$

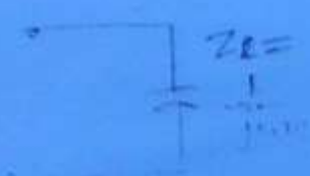
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \quad ; \quad S = \frac{1+\rho}{1-\rho} = \infty = \frac{V_{max}}{V_{min}}$$

$$= \frac{jx - Z_0}{jx + Z_0}$$



High

$$\Gamma = \underbrace{|\Gamma|}_{=\rho} e^{j\theta}$$



low

$$\theta = -2 \tan^{-1}\left(\frac{x}{Z_0}\right)$$

features:

1. if the line is terminated by a purely reactive load then i/p imp. is also purely reactive.
2. The location of voltage max. & Vmin. in the standing wave pattern will depend upon the type of the reactive load.

for inductive load the Vmax will occure at the load end whereas minima will occure at the load end if it is capacitive Nature.

ex: A lossless TL of length l has charstic imp. of Z_0 & is terminated by load imp. Z_L. find input imp. of the line when :-

case:1: $l = \lambda$
case 2: $l = \lambda/2$
case 3: $l = \lambda/4$ ---- QWT Quarter wave transformer
case 4: $l = \lambda/8$

To find :- case : 1 ; $l = \lambda$

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}$$

$$\tan\beta l = \tan\left(\frac{2\pi}{\lambda}\cdot\lambda\right) = 0$$

$$Z_{in} = Z_0 \frac{Z_L + 0}{Z_0 + 0}$$

$$\boxed{Z_{in} = Z_L}$$

Case 2: $l = \lambda/2$

$$\tan\beta l = \tan\left(\frac{2\pi}{\lambda}\cdot\frac{\lambda}{2}\right) = 0$$

$$\boxed{Z_{in} = Z_L}$$

Case : 3 $\quad l = \lambda/4$

QWT

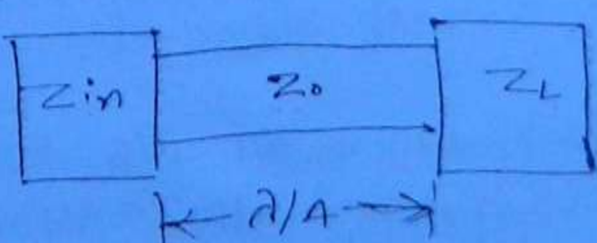
Quarter wave transformer

$$\tan\beta l = \tan\left(\frac{2\pi}{\lambda}\cdot\frac{\lambda}{4}\right) = \infty$$

$$Z_{in} = Z_0 \frac{\cancel{(Z_L/\tan\beta l)} + jZ_0}{\cancel{(Z_0/\tan\beta l)} + jZ_L}$$

$$\boxed{Z_{in} = \frac{Z_0^2}{Z_L}}$$

$$\boxed{Z_0 = \sqrt{Z_{in}\cdot Z_L}}$$



(1) A QWT exists an impedance inversal. therefor if the load Z_L is inductive then the i/p impedance is capacitive & vice-versa.
(2) A λ/4 section of the line Matched two impd. Z_L & Z_{in} & perfect matching take place whenever the load Z_L & i/p ~~imp.~~ is purely resistive imp.
(3) A QW section of the line is used to transform given load imp. is Z_L to the desired i/p imp. using a QWT whose charstics imp. is the geometric mean of the load Z_L & input imp Z_L.

24

$\boxed{\text{Case: 4}}$:- $l = \lambda/8$

$$\tan\beta l = \tan\left(\frac{2\pi}{\lambda}\cdot\frac{\lambda}{8}\right) = 1$$

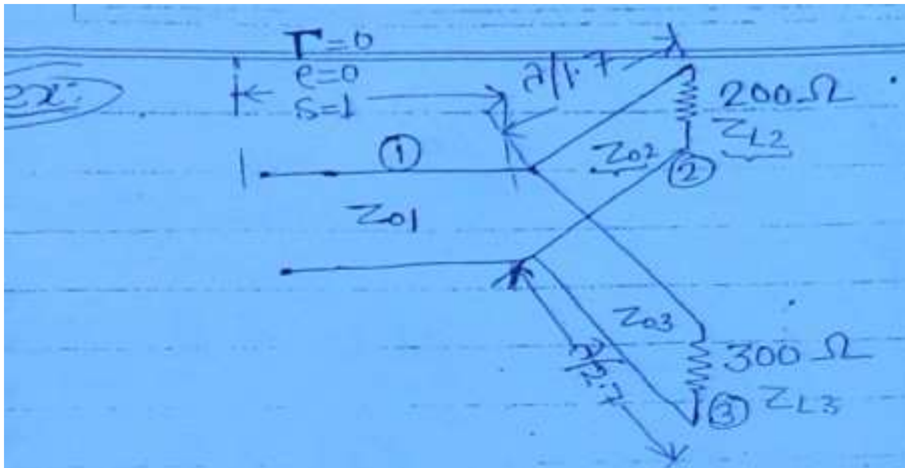
$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \boxed{\tan\beta l}}{Z_0 + jZ_L \boxed{\tan\beta l}} = 1$$

$$\boxed{Z_{in} = Z_0 \frac{Z_L + jZ_0}{Z_0 + jZ_L}}$$

---- complex nature

$$|Z_{in}| = Z_0$$

(1) for λ/8 section of ~~the~~ line the input impedance is always complex in nature irrespective of the nature of load imp Z_L
(2) The magnitude of input impedance of λ/8 section of the line is always as numerically equal to the characteristics of the line.



find the char'ty impedance Z_{01} of the 1st line so that there is no reflected wave on it.

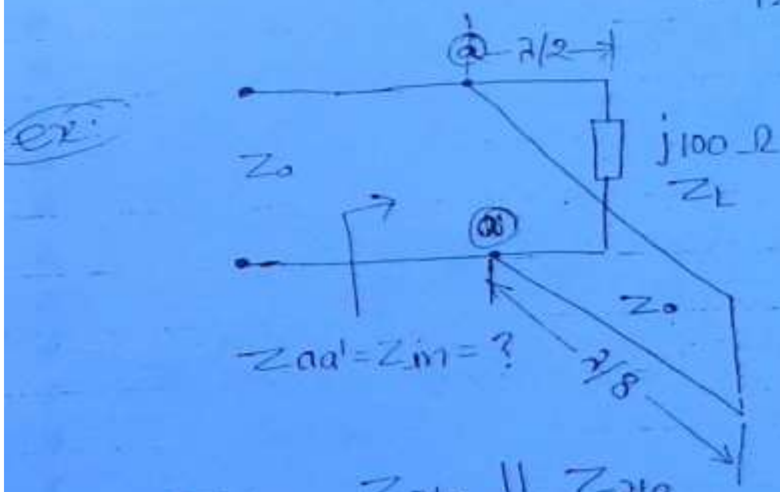
$$Z_{01} = Z_{L1} = Z_{in2} \parallel Z_{in3}$$

-- matched line

$$\left(= Z_{02} \parallel Z_{03} \right)$$

matched line

$$= 200 \parallel 300$$
$$= 120\,\Omega$$



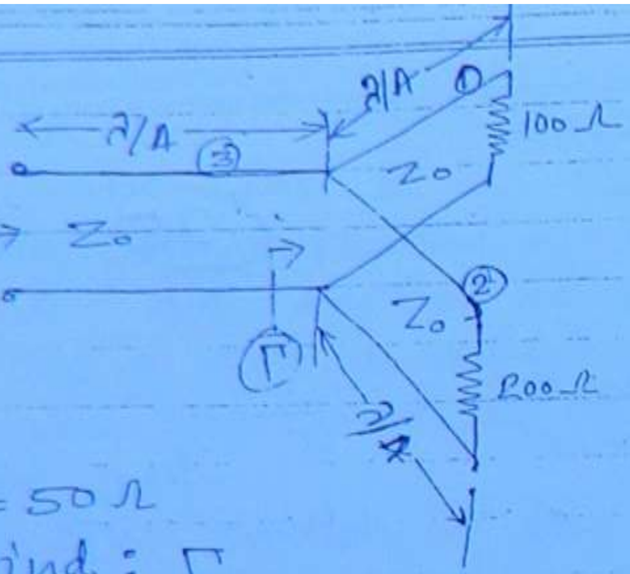
$Z_{aa'} = Z_{in} = ?$
$Z_0 = 200\,\Omega$
to find: Z_{in}

$$Z_{in} = \underbrace{Z_{\lambda/2}}_{\substack{= Z_L \\ = j100\,\Omega}} \parallel \underbrace{Z_{\lambda/8}}_{= jZ_0 \tan \beta l}$$

$$= j200 \cdot \underbrace{\tan\left(\frac{2\pi}{\lambda} \cdot \frac{\lambda}{8}\right)}_{=1}$$

$$Z_{in} = \frac{j100 \times j200}{j100 + j200}$$
$$\frac{j200}{3}\,\Omega$$

ex:



$Z_0 = 50\,\Omega$

to find: Γ

$$Z_{L3} = Z_{in1} \parallel Z_{in2}$$

$$= \frac{Z_0^2}{Z_{L1}} \parallel \frac{Z_0^2}{Z_{L2}} \Rightarrow \frac{2500}{100} \parallel \frac{2500}{200}$$

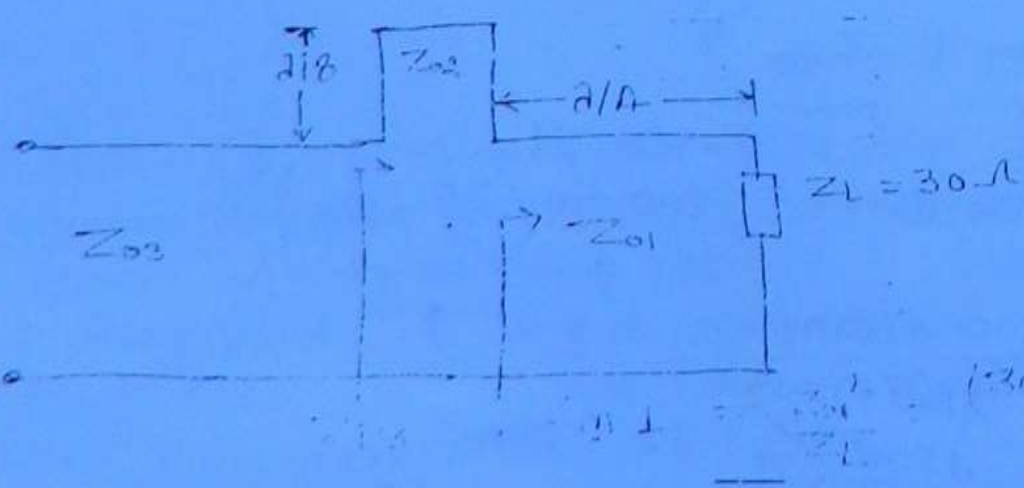
$$\Rightarrow 25 \parallel 25/2 \Rightarrow 25/3\,\Omega$$

$$\Gamma \neq \frac{Z_{L3} - Z_0}{Z_{L3} + Z_0} = \frac{25/3 - 50}{25/3 + 50} = -5/7$$

$$Z_{in} = \frac{Z_0^2}{Z_{L3}} = \frac{2500}{25/3} = 300\,\Omega$$

$$\Gamma = \frac{Z_{in3} - Z_0}{Z_{in3} + Z_0} = \frac{300 - 50}{300 + 50} = +\frac{5}{7}$$ Ans.

ex:



$\frac{(30\sqrt{2})^2}{50}$ 60 Ω

$Z_{01} = 30\sqrt{2}$

$Z_{02} = 30$

$Z_{03} = 60$

To find:-

S on Z_{03} line

$$S = \frac{1+\rho}{1-\rho}$$

$$\rho = |\Gamma|$$

$$\Gamma = \frac{Z_{L3} - Z_{03}}{Z_{L3} + Z_{03}}$$

$$Z_{L3} = Z_{in} = Z_{\lambda/8} + Z_{in1}$$

$$= jZ_{02} \tan \beta l \qquad \tan\left(\frac{2\pi}{\lambda} \cdot \lambda/8\right) = 1$$

$$= j30$$

$$Z_{L3} = j30 + 60$$

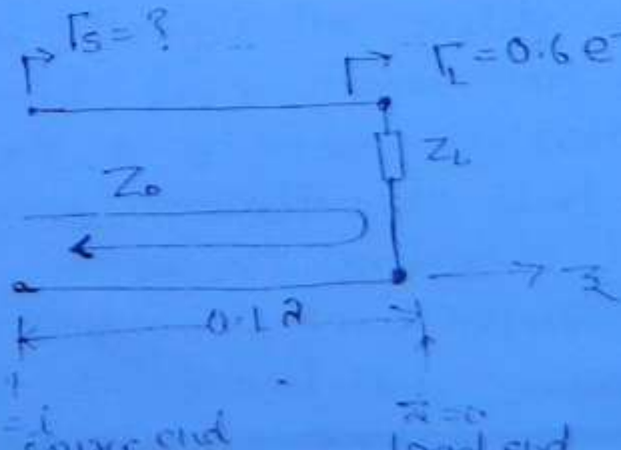
$$\Gamma = \frac{Z_{L3} - Z_{03}}{Z_{L3} + Z_{03}} = \frac{\cancel{60} + j30 - \cancel{60}}{60 + j30 + 60} = \frac{j30}{120 + j30}$$

$$= \frac{j1}{4 + j1}$$

find $\rho = |\Gamma|$

find $S = \dfrac{1+\rho}{1-\rho}$

ex:



$\Gamma_s = \rho_s \cdot e^{j\theta}$
$= 0.6\, e^{j\theta}$

28

$\theta = \underbrace{\phi}_{\uparrow} + \underbrace{(-30^\circ)}_{\text{due to load}}$

due to path difference

$\phi = (\text{path diff.}) \times \underbrace{\frac{2\pi}{\lambda}}_{\beta} \Rightarrow 2z \cdot \frac{2\pi}{\lambda} \Big|_{-z=l}$

$= -2l \cdot \frac{2\pi}{\lambda}$

$= -2 \times 0.1\lambda \times \frac{2\pi}{\lambda}$

$= -0.4\pi$

$= -0.4 \times 180^\circ$

$\phi = -72^\circ$

$\theta = \phi_1 + (-30^\circ)$
$\theta = -72^\circ - 30^\circ$
$\theta = -102^\circ$

$\Gamma_s = \rho_s\, e^{j\theta}$
$= 0.6\, e^{-j102^\circ}$
$\Gamma_s = 0.6\, e^{+j258}$ Ans.

Distortionless line :-

$$\gamma = \alpha + j\beta$$
$$= \sqrt{(R+j\omega L)(G+j\omega C)}$$

29

$$\alpha = f(\omega, \omega^2, \omega^4)$$
$$\beta = f(\omega, \omega^2, \omega^4)$$

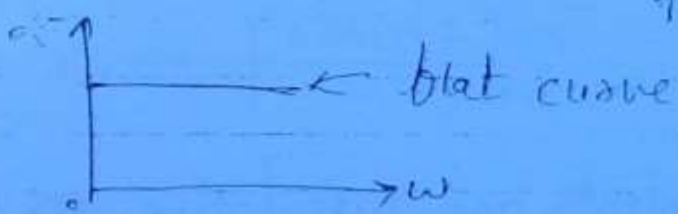
1. Frequency distortion -

$$\boxed{\alpha = f(\omega, \omega^2, \omega^4)}$$

---- causes freq. distortion.

$$\boxed{\alpha \neq f(\omega)}$$

--- to avoid freq. distortion



Equalizer
--- lattice network.

(I) The frequency distortion occurs since various frequency components are subjective to different amount of attenuation.

this changes the quality of the voice signal.

2. To avoid freq. distortion all the frequency components must been subjective to same amount of attenuation so that quality of voice signal remains same.

therefore freq. response attenuation constant, α should be constant & independent

of freq. $f(\omega)$

) praticaly the freq. distortion is avoided by using an equliser which represent a lattice n/w.

phase or delay distortion :-

$$\boxed{\beta = f(\omega, \omega^2, \omega^4)}$$

---- Causes phase or delay distortion

$\beta = \frac{\omega}{V_p}$ must be constant to avoid Dispersion

$\beta \propto \omega$

$$\boxed{\beta = K\omega}$$

1) Various frequency compt are subjective to diff amound of phase shift which causes the phase or delay distortion.

) To Avoid the phase or delay distortion phase const. β must be directly proportional to freq. of operation so that the phase velocity along the line must remain const.

therefor the freq. response of the phase const. β must have linear variation with the frequency variation.

3) Practicall freq. distortion is Avoided by using phase or delay equaliser.

for distortion less line

31

$$\boxed{\begin{array}{l}\alpha \neq f(\omega) \\ \beta = K\omega\end{array}}$$

$\alpha \neq f(\omega)$ - to Avoid freq. distortion.

$\beta = K\omega$ - to Avoid phase of delay distortion.

$$\Rightarrow \boxed{RC = LG}$$ ---- condition for a distortion less line.

* Characteristic impedance of distortion less line:

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$\therefore RC = LG$

$R = \frac{LG}{C}$

$$\Rightarrow Z_0 = \sqrt{\frac{LG/C + j\omega L}{G+j\omega C}} = \sqrt{\frac{L(G+j\omega C)}{C(G+j\omega C)}}$$

$$Z_0 = \sqrt{\frac{L}{C}}$$

same as that of lossless line.

Case 1: at High frequency

$\omega L >> R$

$\omega C >> G$

$$Z_0 = \sqrt{\frac{\cancel{R}+j\omega L}{\cancel{G}+j\omega C}} \cong \sqrt{\frac{L}{C}}$$

Case 2: At low frequency.

$\omega L << R$

$\omega C << G$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} \cong \sqrt{\frac{R}{G}}$$

(32)

$$\because RC = LG$$

$$Z_0 \cong \sqrt{\frac{L}{C}} \qquad \frac{R}{G} = \frac{L}{C}$$

(1) The charsti's gmp. of a lossless & distortionless line is same & depends only upon the primary constant L & C.

(2) At High freq. the TL will always behaves as a lossless as well as distortion less line irrespective of the condition RC = LG is satisfied or NOT.

(3) At low frequency the line in general does not behaves as a distornless line unless the condition RC = LG is satisfied.

Summary :-

Distortionless line	lossless line
$\boxed{\alpha \neq f(\omega)}$	$R = G = 0$
$\beta = k\omega$	$\boxed{\alpha = 0}$
$\beta = \frac{\omega}{V_p}$	$\beta = \frac{\omega}{V_p}$
$V_p = \frac{1}{\sqrt{LC}}$	$V_p = \frac{1}{\sqrt{LC}}$
$Z_0 = \sqrt{\frac{L}{C}}$	$Z_0 = \sqrt{\frac{L}{C}}$
$\boxed{RC = LG}$	

Conclusion :- ① for a distortion less line α must be independent of freq. but may have a finite value.

There for in general a distortion line is a lossy line.

33

② for a lossless line R & G are zero & there for the condition RC = LG is always satisfied.

Since α = 0 for such line it's independent of freq. ω.

there for any loss less line is always a distortion less line.

③ pratically any transmission line is always a lossy line & there for cannot be a distortion less line.

Such line may be made distortion less by suitably selecting the material such that the condition RC = LG is always satisfied.

Ex:
$Z_0 = 50\,\Omega$
$R = 0.1\,\Omega/m$

To find α for distortionless line.

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$\frac{L}{C} = Z_0^2 = 2500$$

$$RC = LG \Rightarrow G = R \cdot \frac{C}{L} = \frac{0.1}{2500}\ S/m$$

$$V = V^+ e^{-j\beta z} + V^- e^{+j\beta z}$$

$$I = \frac{1}{Z_0}\left(V^+ e^{-j\beta z} - V^- e^{+j\beta z}\right)$$

$$V\Big|_{-z=l} = V_S = V^+ e^{+j\beta l} + V^- e^{-j\beta l}$$

$$e^{\pm j\beta l} = \cos\beta l \pm j\sin\beta l$$

$$\frac{V^-}{V^+} = \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \; ; \; Z_L = \frac{V_L}{I_L}$$

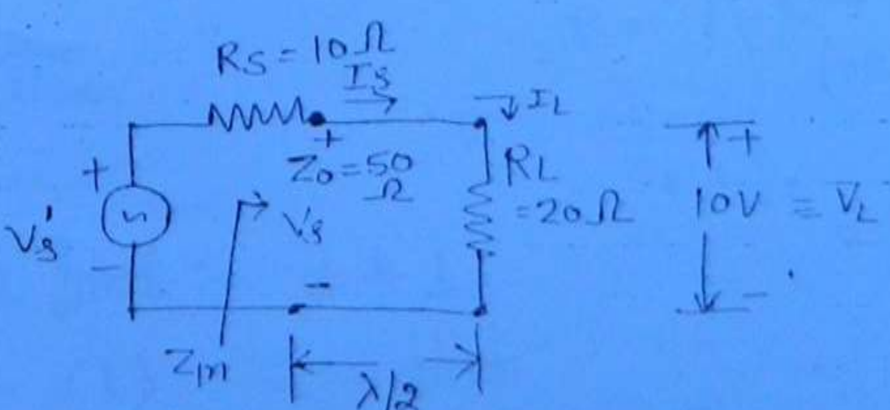
$$V^+ \Rightarrow \frac{V_L}{V^+} = T \; ; \; V^+ = \frac{V_L}{T}$$

$$T = \frac{2Z_L}{Z_L + Z_0} \; ; \; Z_L = \frac{V_L}{I_L}$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \cos\beta l & j Z_0 \sin\beta l \\ (j/Z_0)\sin\beta l & \cos\beta l \end{bmatrix} \begin{bmatrix} V_L \\ I_L \end{bmatrix}$$

$$\begin{bmatrix} V_L \\ I_L \end{bmatrix} = \begin{bmatrix} \cos\beta l & -j Z_0 \sin\beta l \\ (-j/Z_0)\sin\beta l & \cos\beta l \end{bmatrix} \begin{bmatrix} V_S \\ I_S \end{bmatrix}$$

ex:



To find: $\begin{cases} V_S ; I_S \\ I_1 ; Z_{in} ; V_S' \end{cases}$

$\alpha \neq f(\omega)$

$\gamma = \sqrt{(R + \cancel{j\omega L})(G + \cancel{j\omega C})} = \alpha + j\beta$

$\alpha \neq f(\omega)$

$\gamma = \alpha + j\beta = \sqrt{RG}$

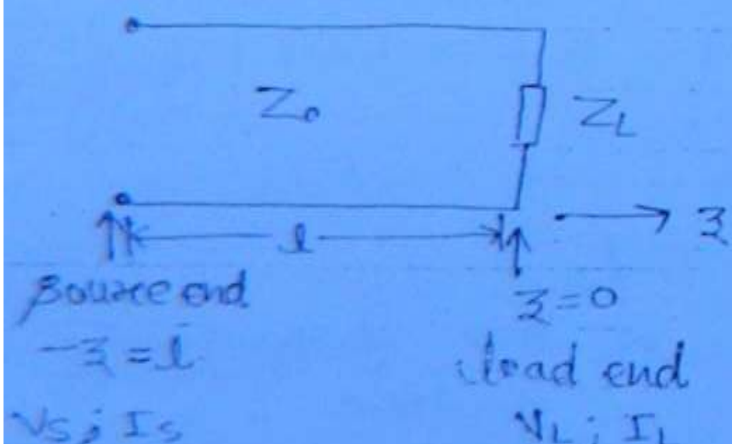
$\alpha = \sqrt{RG} = \sqrt{0.1 \times \frac{0.1}{2500}}$

$\alpha = 0.002$ nepers/m

$\boxed{1 \text{ neper} = 8.686 \text{ dB}}$

$\alpha = 0.002 \times 8.686$
--- dB

Transmission matrics :-



$\left.\begin{matrix} V_s \\ I_s \end{matrix}\right\} \longleftrightarrow \left\{\begin{matrix} V_L \\ I_L \end{matrix}\right.$

For a lossless line.

$$I_L = \frac{V_L}{R_L} = \frac{10}{20}$$

36

$$I_L = \frac{1}{2} \text{ Amp.}$$

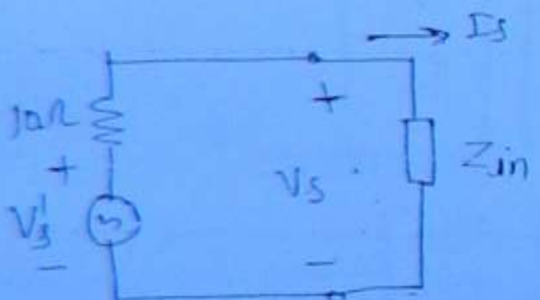
$$V_s = \underbrace{\cos\beta l}_{\cos\left(\frac{2\pi}{\lambda}\cdot\frac{\lambda}{2}\right)} \cdot \underset{10}{V_L} + j\, \underset{50}{Z_0}\, \underbrace{\sin\beta l}_{\sin\left(\frac{2\pi}{\lambda}\,\lambda/2\right) = 0} \cdot I_L \leftarrow 1/2$$

$$\boxed{V_s = -10V}$$

Simple method for I_s

$$Z_{in} = Z\big|_{\lambda/2} = Z_L = 20\,\Omega$$

$$\boxed{Z_{in} = 20\,\Omega}$$



$$I_s = \frac{V_s}{Z_{in}} = \frac{-10}{20} = -1/2\,A$$

$$V_s' = I_s(10+20)$$

$$V_s' = -\frac{1}{2}(10+20)$$

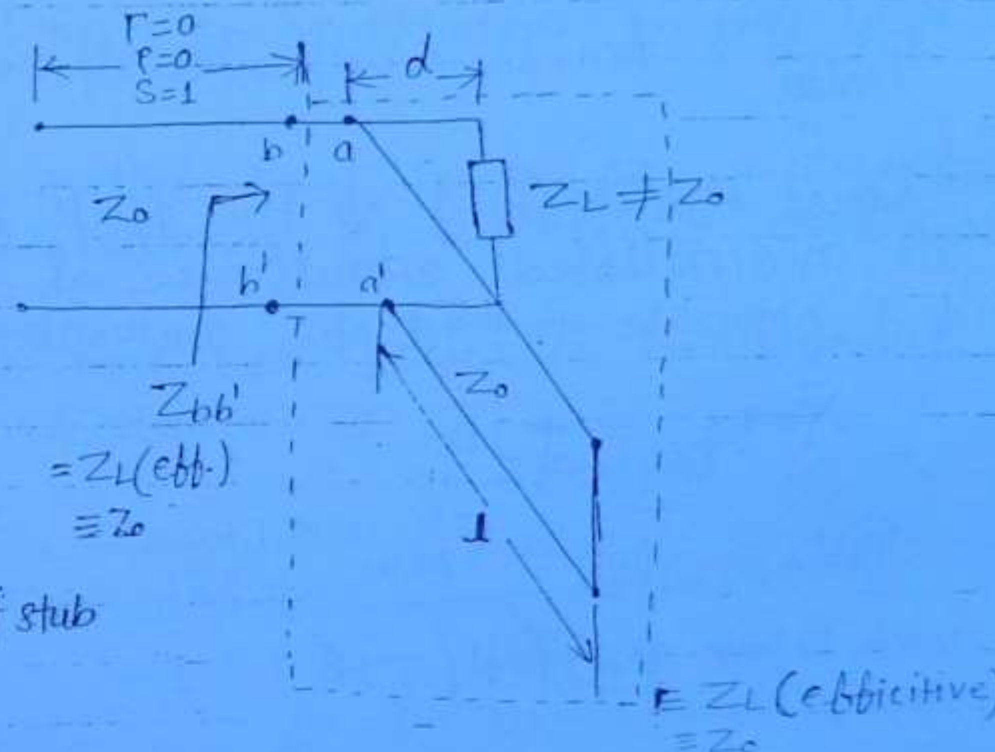
$$\boxed{V_s' = -15\,V}$$

Stub Matching :-

.... Required when $Z_L \neq Z_0$

(37)

Case 1 :- Short circuit shunt stub :-



where :-
d -- location } of stub
l = length of } of stub

$S_{\text{Main line}} = 1$

$S_{\text{stub line}} = \infty$

$S_{\text{load line}} = ?$

$$S = \frac{1+\rho}{1-\rho}$$

$$\rho = |\Gamma|$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

to find :- d & l

(1) Z_L = given.

(2) find $\left(\frac{Z_L}{Z_0}\right) = \bar{Z}_L$

(3) find $\bar{Y}_L = \frac{1}{\bar{Z}_L}$

(4) Move a distance 'd' such that

$$\overline{Y}_{aa'} = 1 \pm j\overline{B}$$

38

Comments : The location of the stub till adjust their normalised value of Real part of admittance at 'aa'' becomes a unity.

(5) adjust the length l of the stub so that it's normalised admittance at aa' is equal & opposite to that of imaginary part of

$$\overline{Y}_{stub} = \mp j\overline{B}$$

(6) $\overline{Y}_{bb'} = \overline{Y}_{aa'} + \overline{Y}_{stub}$

$$= 1 \pm \cancel{j\overline{B}} + (\mp j\overline{B}) = 1$$

(7) $\overline{Z}_{bb} = \dfrac{1}{\overline{Y}_{bb'}} = 1$

(8) $\dfrac{Z_{bb'}}{Z_0} = 1$; $Z_{bb'} = Z_0$

$= Z_L$ (effectively)

(9) Hence the TL is perfectly matched with the effectively load imp. at bb'

(10) therefor to the left side of bb' the line is perfectly matched, there is no reflected waves, no standing wave pattern, reflection coefficient is zero & therefor has a

Minimum value of VSWR is unity.
therefor max. power is transferd from the source to load.
the stub line will act as only the reactive power since the o/p imp. of stub is purely reactive.
this Reactive power is not a useful power.
the entire Real power or the useful power is transmitted to the load impedance Z_L.
Hence max. Real power is been transfere to load impedance Z.

Ex:
$Z_L = 100 + j300\,\Omega$ → Real part
$Z_0 = 100\,\Omega$ → Real part
Both are equal then normilised is equal to 1

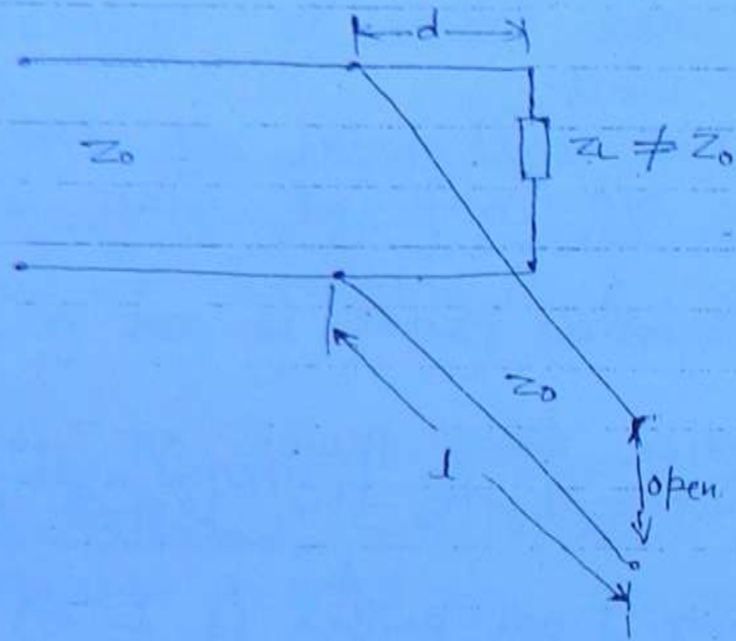
so $d = 0$
Stub is connected at the load.

Ex:
$Z_L = 200 + j300$
$Z_0 = 100$

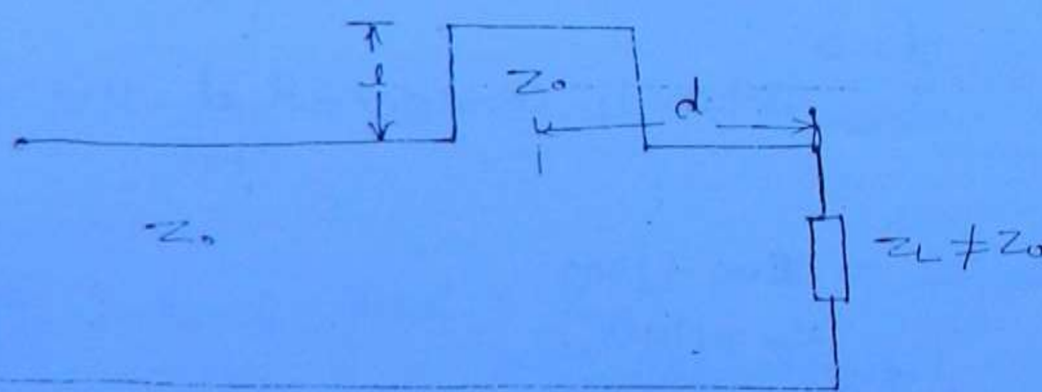
$d \neq 0$
stub is connected at some specific Distance from the load.

(1) The stub Matching is used only by a short circuit or open circuit stub line.
discreate component's of L & C are not used for stub matching.

case-2 :- open circuit shunt stub :-

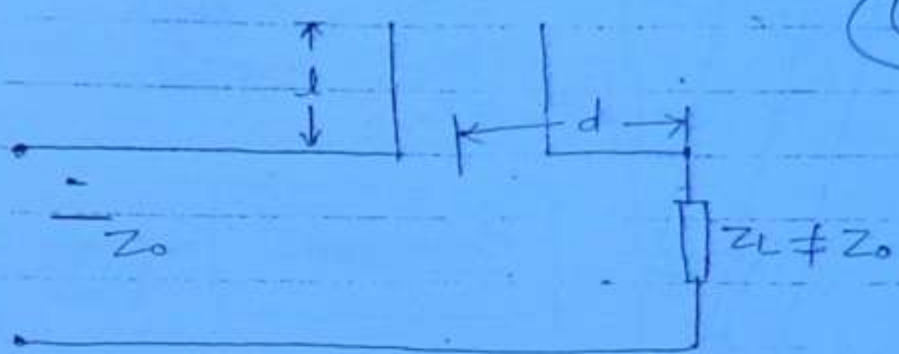


case: 3 - SC series stub :-



case: 4 O.C series stub:-

41



① The s.c stub is always prefferred since the adjustment of the length is more conventional pratically.

② the o.c. stub is normally not preffred since

ⓐ the adjustment of the length is pratically not convention.

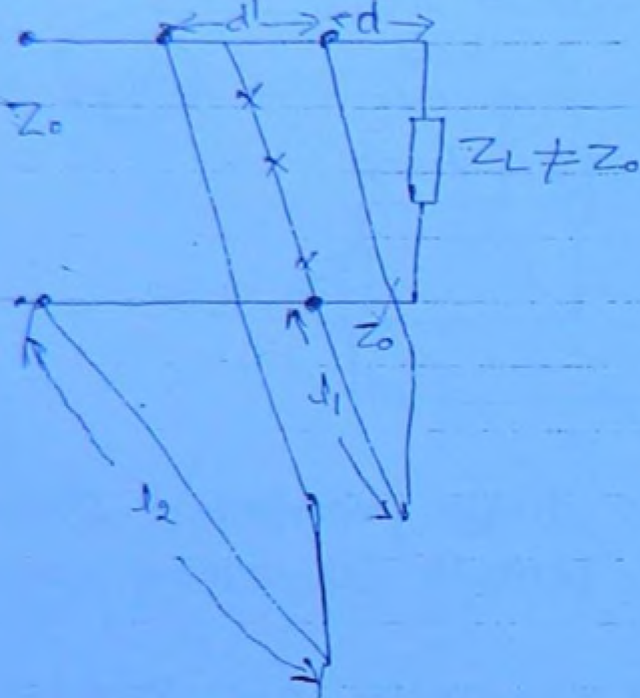
ⓑ an O.C. Stub X has a entina & em. po^r is radiated from it.

③ The shunt stub is always prefferd since the main line remains unaffected when the load is varied over wide range.

④ the series stub is never Preffered since the main line is affected if the load Z_L is varied over a wide range.

⑤ Therefor for variable load the S-C shunt stub Matching is the best where as O.C se stub Matching is the west

Double Stub matching



① Double stub matching is generally peffered over the single stub matching because of more flexibility in the variation of length of each stub l_1 & l_2.

using double stub matching we are not able to match all the type of load with the charatstic imp. of the line

Variation of impedance along the line

$$Z_{max} = \frac{V_{max}}{I_{min}} = \frac{V_{max}}{V_{min}/Z_0} = Z_0 . S$$

$$\left(\frac{Z_{max}}{Z_0}\right) = \boxed{\bar{Z}_{max} = S}$$

$$Z_{min} = \frac{V_{min}}{I_{max}} = \frac{V_{min}}{V_{max}/Z_0} = \frac{Z_0}{S}$$

$$\left(\frac{Z_{min}}{Z_0}\right) = \boxed{\bar{Z}_{min} = \frac{1}{S}}$$

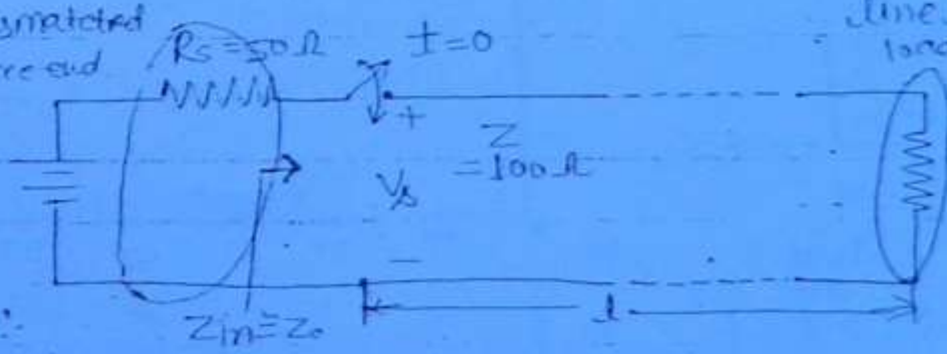
$$\boxed{\frac{1}{S} \le \bar{Z} \le S}$$

Transient Response in T.L. :-

line is mismatched at the source end

line is mismatched at load end ($Z_L \neq Z_0$)

$V_S' = 12V$



$Z_L = 200\,\Omega$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{200 - 100}{200 + 100} = +1/3$$

assumption :-

1. line is lossless.
2. l is large $\quad Z_{in} \simeq Z_0 = 100\,\Omega$
3. $T = l/v_p$ ..... time taken by voltage waveform to reach from source to load end.
4.

$T \sim 400\ \mu sec$

$\quad - 500\ \mu sec.$

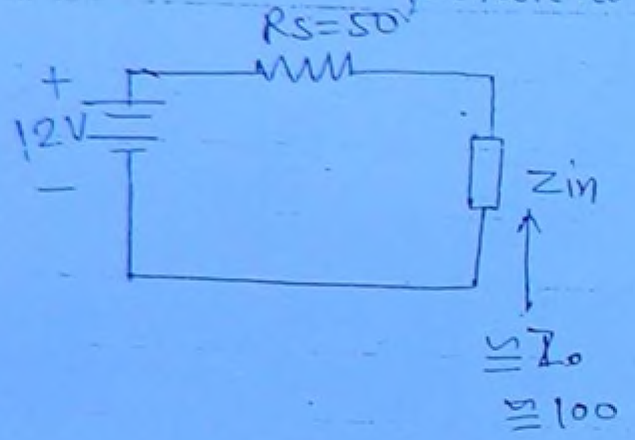
$$\Gamma_S = \frac{R_S - Z_0}{R_S + Z_0} = \frac{50-100}{50+100} = -\frac{1}{3}$$

44

To find:

$v(t)$ vs t

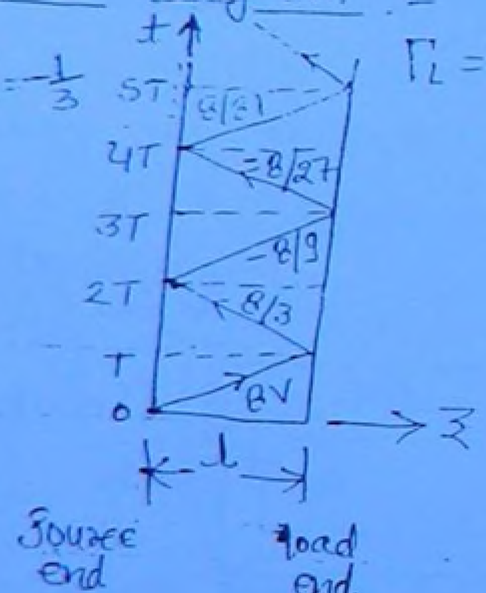
---- transient response of the line.

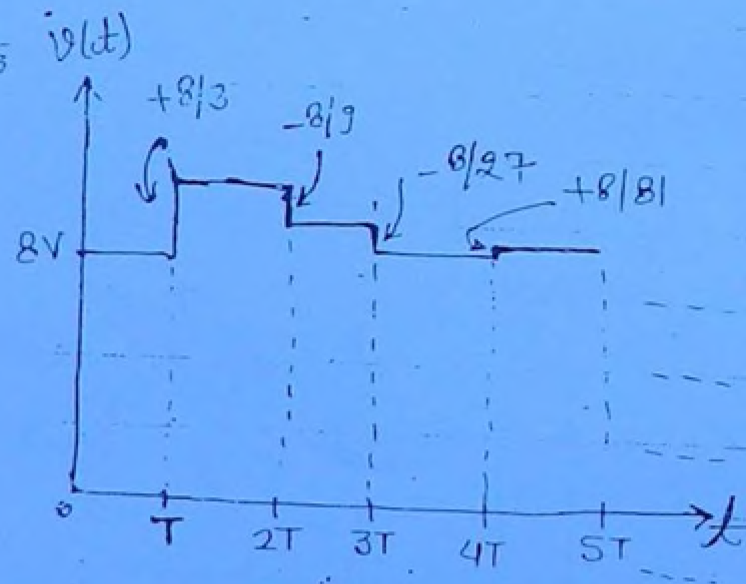


$$V_S = \frac{100}{50+100} \times 12$$
$$= \frac{2}{3} \times 12$$
$$= 8V$$

Bounce diagram:-

$\Gamma_S = -\frac{1}{3}$ $\qquad$ $\Gamma_L = +\frac{1}{3}$





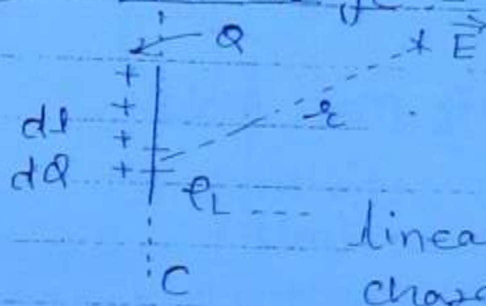
Sources of $\vec{E}$ (electrical field) :-

① Point charge :-

$\vec{E} \propto \frac{1}{r^2}$

Coulomb's law

② Line charge :-



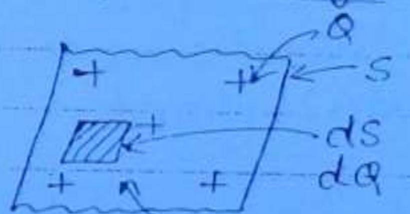
ρ_L --- linear charge density

charge per unit length (C/m)

$dQ = \rho_L \cdot dl$

$$\boxed{Q = \int_C \rho_L \, dl}$$

③ Surface charge :

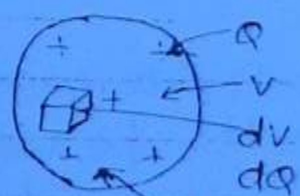


ρ_S --- surface charge density (C/m^2)

$dQ = \rho_S . ds$

$$\boxed{Q = \iint_S \rho_S . ds}$$

④ Volume charge



ρ_v; ρ --- Volume charge density (C/m^3)

$dQ = \rho . dv$

$$\boxed{Q = \iiint_V \rho . dv}$$

Source of $\vec{B}$ (Mag. field)

1) line current :-

$\vec{B} \propto \frac{1}{r^2}$ ---- Biot-savart's law

dl, r, (A)

$I = I_0 \Rightarrow \vec{B} = \vec{B}_0$ static field

$= I_0 \sin\omega t$

C $\Rightarrow \vec{B} - \vec{B}_0 \sin\omega t$ ---- time varying mag. field

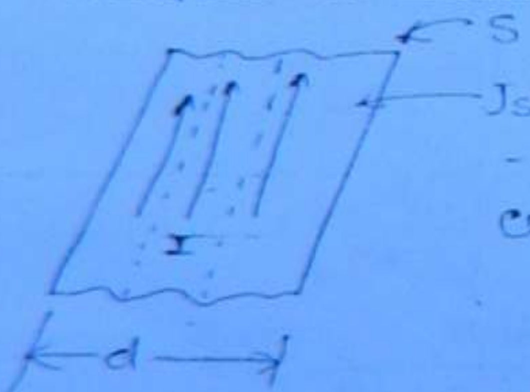
current element

$$\boxed{\vec{I}\, dl = I\, d\vec{l}}$$

convention :-

by convention the direction of current I & elementary length dl are taken same in all electromag. problems.

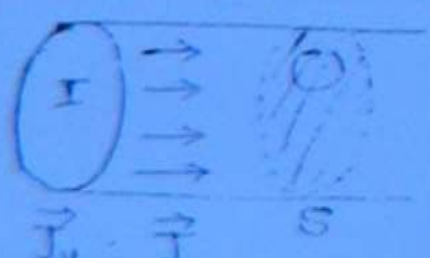
2) Surface current :-



--- Surface current density (A/m)

$$\boxed{I = \int_d J_s \cdot dl}$$

3) Volume current :-



$\vec{J}_v : \vec{J}$

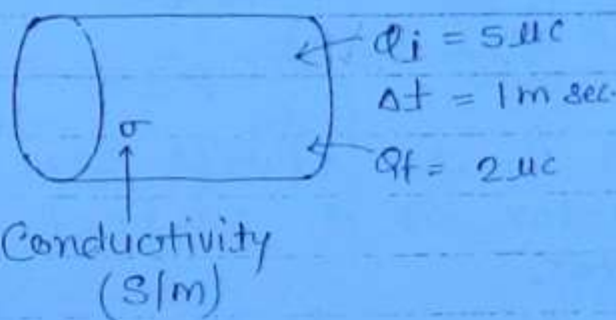
Volume current density (A/m^2)

$$\boxed{I = \iint_S \vec{J} \cdot d\vec{s}}$$

Continuity eqn.

---- Conversion of charge (47)

$$\boxed{I = -\frac{dQ}{dt}}$$



$$I = -\frac{dQ}{dt} = -\frac{Q_f - Q_i}{\Delta t}$$

$$= -\frac{(2-5)\times 10^{-6}}{1\times 10^{-3}}$$

$$= 3mA$$

$$\rightarrow \boxed{\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}}$$

Volume charge density (C/m^2) ← ρ

Volume current density (A/m^2) ← $\vec{J}$

----- continuity eqn.

Where $\boxed{\vec{J} = \sigma \vec{E}}$ --- ---- Ohm's law

if $\rho = 0$; $\frac{\partial \rho}{\partial t} = 0$

$$\rightarrow \boxed{\nabla \cdot \vec{J} = 0}$$

... $\vec{J}$ is solenoidal in nature.

.... KCL equns.

Statement :- (1) The diversions of volume current density $\vec{J}$ at any point on the electromagnetic Region is always equal to the rate of decrease of the volume charge density ρ with respect time t.

there for there is a continuity b/w the decrement of volume charge density row & the correspoing production volume current density $\vec{J}$.

(2) for a charge free Region the diversion of volume current density $\vec{J}$ at any point is always equal to zero

and hence volume current density is always solenoidal & forms a closed loop.

48

$\rho = 0$

--- on perfect cond ($\sigma \simeq \infty$)

--- on dielect cond. ($\sigma \simeq 0$)

① The charge density ρ is always equal to zero on a perfect conductor or a perfect dielectric conductor.

② Volume charge density is finite on a medium where the conductivity is finite.

the decay of volume charge density will depend upon the conductivity of region.

Higher is the conductivity higher is the rate decay of charge & vice-versa.

Maxwell's eqns. in their general time varying form :-

differential form (point form)	integral form
$\nabla \cdot \vec{D} = \rho$	$\underbrace{\oiint_S \vec{D} \cdot d\vec{s}}_{\text{electric flux}} = Q = \iiint_V \rho\, dv$ Gauss's law for elect. fields
$\nabla \cdot \vec{B} = 0$	$\oiint_S \vec{B} \cdot d\vec{s} = 0$ --- Gauss's law for [illegible] fields

(3.) $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$\underbrace{\oint_C \vec{E} \cdot d\vec{l}}_{e.m.f} = -\iint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$ (49)

faraday's law of e.m. induction.

$\nabla \times \vec{H} = \vec{J_c} + \vec{J_d}$

$= \sigma \vec{E} + \frac{\partial \vec{D}}{\partial t}$

$\underbrace{\oint_C \vec{H} \cdot d\vec{l}}_{m.m.f} = \iint_S (\vec{J_c} + \vec{J_d}) \cdot d\vec{S}$

--- Modified Ampere's circuital law

---- eqn. using displacement current density concept

$\vec{J_c} = \sigma \vec{E}$

--- Cond. current density (A/m^2)

$\vec{J_d} = \frac{\partial \vec{D}}{\partial t}$ -- displacement current density (A/m^2)

Statements :-

(1) (1a) The diversions of electrical flux density $\vec{D}$ at any point in the electro mag. Region is always equal to the volume charge density ρ.

(1b): The net electrical flux passing through any closed surface area S is alway equal to total charge enclose within the surface Area S.

(2) 2(a) The diversions of mag. flux density $\vec{B}$ is always equal to zero since

Reasons: (i) Mag. field line are always closed in nature.

(ii) The Mag. charges in the isolated form do not exists in nature.

2(b) The net mag. flux passing through any closed surface Area S is always equal to zero.

50

3) 3(a) : The ~~curve~~ curl of elec. field intensity $\vec{E}$ is ~~eq~~ always equal to the rate of decrease of mag. flux density $\vec{D}$ w.r.to time t.

3(b) Net emf produced is always equal to the surface integral of rate of decrease of mag. flux density $\vec{B}$ w.r.to time t.

4(a) The curl of mag. field intensity $\vec{H}$ is always equal to the some of conduction current density $\vec{J_c}$ & the displacement current density $\vec{J_B}$.

4(b) Total mmf produced is always equal to the surface integral of the some of conduction current density $\vec{J_c}$ & displacement ~~cu~~ density $\vec{J_B}$

Special cases:

Case 1: for static fields:-

$$\boxed{\begin{aligned} \frac{\partial \vec{B}}{\partial t} &= 0 \\ \frac{\partial \vec{D}}{\partial t} &= 0 \end{aligned}}$$

Case 2 :- for perfect diel ($\sigma = 0$)

or

non-Conducting Meadium

or

lossless meadium

or

free space

(51)

$$\boxed{\begin{aligned} \vec{J_C} &= \sigma \vec{E} = 0 \\ \rho &= 0 \end{aligned}}$$

Case 3 :- for Good Conductor ---- σ is High

$$\boxed{\begin{aligned} \vec{J_B} &\simeq 0 \\ \rho &\simeq 0 \end{aligned}}$$

1. for time-Harmonically OR sinusoidally varying fields :-

$$\vec{D} = \vec{D_0} e^{j\omega t}$$
$$\vec{B} = \vec{B_0} e^{j\omega t}$$

$$\frac{\partial \vec{B}}{\partial t} = j\omega \underbrace{\vec{B_0} e^{j\omega t}}$$

$$= j\omega \vec{B}$$

$$\boxed{\begin{aligned} \frac{\partial}{\partial t} &\rightarrow j\omega \\ \frac{\partial^2}{\partial t^2} &\rightarrow (j\omega)^2 = -\omega^2 \end{aligned}}$$

So. Eqns:

(1) $\nabla \cdot \vec{D} = \rho$

(2) $\nabla \cdot \vec{B} = 0$

(3) $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -j\omega\mu\vec{H}$

(4) $\nabla \times \vec{H} = \sigma\vec{E} + \frac{\partial \vec{D}}{\partial t}$

$= \sigma\vec{E} + j\omega\epsilon\vec{E}$

$= (\sigma + j\omega\epsilon)\vec{E}$

★. loss tangent ($\tan\delta$)

52

$$\vec{J}_{total} = \vec{J}_c + \vec{J}_d$$

$$= \sigma\vec{E} + \frac{\partial\vec{D}}{\partial t}$$

$$\cong \sigma\vec{E} + j\omega\epsilon\vec{E}$$ ---- for general meadium (μ, ϵ, σ).

$$\cong \sigma\vec{E}$$ ---- for good conductor.

$$\cong j\omega\epsilon\vec{E}$$ --- for good dieelectric.

$$\tan\delta = \frac{|\vec{J}_c|}{|\vec{J}_d|} = \frac{|\sigma\vec{E}|}{|j\omega\epsilon\vec{E}|} = \frac{\sigma E}{\omega\epsilon}$$

$$\boxed{\tan\delta = \frac{\sigma}{\omega\epsilon} \begin{array}{l} >>1 \\ <<1 \end{array}}$$ --- Good conductor. ---- Good dieelectric.

imp points:

1) for Good conductors the conductivity is high & therefor the conduction current density is domanity.

2) for Good diel. the conductivity is low, conduction current density is neglible & the displacemat current density is more dominated.

3) the loss tangent is the ratio of the magnitude of ~~current~~ conduction current density & displacement current density

this is a major of total loss occuring in a material due to finite conductivity as peeified frequency.

(4) depending upon the frequency of operation any meadium may behaves as a good conductor

or good dielectric.

In general any material may behaves as a good conductor at low freq. whereas same material may behaves as a good dielectric at high frequency.

Conclusion :

Therefor depending upon Application we operate a device at high freq. and low freq. so that it can operate at a good dielectric & good conductor.

Poynting's vector ($\vec{P}$)

$$\boxed{\vec{P} = \vec{E} \times \vec{H}} \quad \dots \frac{V}{m}, \frac{A}{m}$$

.... W/m^2

- Power density at a point

$$\text{power} = \iint_s \vec{P} \cdot d\vec{s}$$

if $\underbrace{(\vec{E} \perp \vec{H}) \perp \vec{P}}_{\text{Transverse e.m. wave}}$

TEM wave → uniform plane wave (Plane wave

$$\vec{P} = \vec{E} \times \vec{H} = EH \underbrace{\sin\theta}_{\sin\theta = 1} . \hat{a}_n$$

$$|\vec{P}| = \boxed{P = EH}$$

$$= E_{rms} \cdot H_{rms}$$

$$\frac{E}{H} = \eta$$ ---- Intrinsic impedance of the Medium.

$$\frac{V/m}{A/m} = \Omega$$

$$\frac{E}{H} = \boxed{\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}}$$ ... for general lossy Meadium (μ, ϵ, σ)

$$\boxed{\eta = \sqrt{\frac{\mu}{\epsilon}}}$$ --- for lossless meadium ($\sigma = 0$)

$$\boxed{\eta = \sqrt{\frac{\mu_0}{\epsilon_r}}} \cong 120\pi \cong 377\ \Omega$$

--- for free space

$\sigma = 0$

$\mu = \mu_0$

$\epsilon = \epsilon_0$

$$|\vec{P}| = \boxed{\begin{aligned} P &= E \cdot H \\ &= \eta H^2 \\ &= \frac{E^2}{\eta} \end{aligned}}$$ --- E; H rms Value

$$\boxed{\begin{aligned} P &= \frac{1}{2} E_m H_m \\ &= \frac{1}{2} \eta H_m^2 \\ &= \frac{1}{2} \frac{E_m^2}{\eta} \end{aligned}}$$

Imp. points :- (1) The poynting vector (P) represent power density at a point.

When integrator over any closed surface Area the total power flow on the specified direction.

The P vectors ~~gay~~ gives the direction of propagation of em waves & is always perpendicular to the plane made by $\vec{E}$ & $\vec{H}$ vectors

(4) for a transfer e.m. waves $\vec{E}$, $\vec{H}$ & $\vec{P}$ are mutually prependicular to each other.

(5) In any e.m. region the ratio b/w the $\vec{E}$ & $\vec{H}$ fields is always constant. & is represent by intrinsic impedance of meadium.

(6) This imp. depends only upon the const. of the Meadium.

for a free space this imp. has a universty const. value of $120\pi\ \Omega$ or $377\ \Omega$ (55)

Ex: An e.m. waves is travelling along $-y$ direction & has only x-comts of elect. fields.

find the magnetic field intensity associted with e.m ways.

$$\vec{P} = \vec{E} \times \vec{H}$$

$$\vec{P} = -P_y \hat{a}_y \text{ ---- direction of prop. of wave.}$$

$$\vec{E} = E_x \hat{a}_x$$

To find :- $\vec{H} = ?$

$$-\hat{a}_y \begin{pmatrix} \hat{a}_x \\ \hat{a}_y \\ \hat{a}_z \end{pmatrix} \begin{matrix} = +\hat{a}_z \\ = +\hat{a}_x \end{matrix} \quad \text{(cross product)}$$

$$\vec{P} = \vec{E} \times \vec{H}$$

$$(-P_y \hat{a}_y) = (E_x \hat{a}_x) \times (+H_z \hat{a}_z)$$

$$\boxed{\vec{H} = +H_z \hat{a}_z}$$

Ex: find the displacement current at $t=0$ through a 10pf capacitor if the voltage across it is given by

$$v(t) = 0.1 \sin 120\pi t \text{ --- V}$$

$$C = 10pf$$

$$I_d|_{t=0} = ? = J_d \cdot A.$$

$$J_d \cdot A = A\frac{\partial D}{\partial t} = \epsilon A \frac{\partial E}{\partial t} = \underbrace{\frac{\epsilon A}{d}}_{C} \frac{\partial}{\partial t}[v(t)]$$

$$C\frac{\partial}{\partial t}(0.1\sin 120\pi t) = \underbrace{10\times 0.1\times 120\pi}_{10\times 12\pi} \underbrace{\cos 120\pi t}_{=1}\Big|_{t=0}$$

$$\cong 120\pi \ \text{pA}$$
$$\cong 377 \ \text{pA}$$
$$\cong 0.377 \ \text{nA}$$

56

Ex: An e.m. wave is travelling in a lossless meadium is $\mu_r = 1$ & $\epsilon_r = 4$ & has a power density of $4\ W/m^2$.

Calculate the max. value of E & H phase.

$|\vec{P}| = P = 4\ W/m^2$

$\mu_r = 1;\ \mu = \mu_0\mu_r = \mu_0$

$\epsilon_r = 4;\ \epsilon = \epsilon_0\epsilon_r = 4\epsilon_0$

To find:

$E_m;\ H_m$

$$P = \frac{1}{2}E_m \cdot H_m \qquad \eta = \frac{E_m}{H_m}$$

$$= \frac{1}{2}\eta H_m^2$$

$$= \frac{1}{2}\frac{E_m^2}{\eta} \leftarrow \qquad \eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0}{4\epsilon_0}} = \frac{\eta_0}{\sqrt{\epsilon_r}}$$

$$= \frac{120\pi}{2} = 60\pi\ \Omega$$

$$P = \frac{1}{2}\frac{E_m^2}{\eta}$$

$$\boxed{E_m = \sqrt{2\eta P}} = \sqrt{2\times 60\pi\times 4}\ V/m$$

$$\eta = \frac{E_m}{H_m};\ \frac{E_m}{60\pi} = H_m\ A/m$$

Wave eqns.

---- Mathematical form of e.m. wave

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{--- (1)}$$

$$\nabla \times \vec{H} = \sigma \vec{E} + \frac{\partial \vec{D}}{\partial t} \quad \text{--- (2)}$$

$$\begin{tikzcd}
\vec{B}(t) \arrow[r, Rightarrow] & \frac{\partial \vec{B}}{\partial t} \arrow[r, "\text{eqn.(1)}"] & \vec{E}(t) \arrow[r] & \vec{D}(t) \arrow[dl] \\
& \vec{H}(t) \arrow[ul] & \frac{\partial \vec{D}}{\partial t} \arrow[l, "\text{eq.(2)}"'] &
\end{tikzcd}$$

$$\begin{tikzcd}
\vec{E}(t) \arrow[r, bend left] & \vec{H}(t) \arrow[l, bend left]
\end{tikzcd}$$

$$\begin{tikzcd}
w_e = \frac{1}{2}\epsilon E^2 \arrow[r, bend left] & \frac{1}{2}\mu H^2 = w_m \arrow[l, bend left]
\end{tikzcd}$$

$$\begin{tikzcd}
\frac{\partial}{\partial t}(w_e) \arrow[r, bend left] & \frac{\partial}{\partial t}(w_m) \arrow[l, bend left]
\end{tikzcd}$$

$$\begin{tikzcd}
P_e \arrow[r, bend left] & P_m \arrow[l, bend left]
\end{tikzcd}$$

$\Downarrow$

power flows in a direction

$\Rightarrow$ e.m. wave propagates.

Conclusion:

(1) due to time varying ele. & mag. field the rate of change of ele. energy is transformed to the rate of change of mag. energy.

& vice - versa.

2) Due to this rate of change the pow propagates in a particular direction which is given by poynting vector (P)

3) there for e.m. waves propagates in the direction given by poynting vector (P)

date - 28-07-2010

$\log_e\left(\frac{V_o}{V_i}\right)$ ... nepers/m

58

$20\log_{10}\frac{V_o}{V_i}$ ... dB

$10\log_{10}\frac{P_o}{P_i}$ ... dB

Wave eqns

equ. — ①

eqns. — ②

eliminate $\vec{H}$

$$\boxed{\nabla^2\vec{E} = \underbrace{\mu\sigma\frac{\partial\vec{E}}{\partial t}}_{\text{loss factor}} + \underbrace{\mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2}}_{\text{propagation factor}}}$$

... wave eqns in $\vec{E}$

Similarly

$$\nabla^2\vec{H} = \mu\sigma\frac{\partial\vec{H}}{\partial t} + \mu\epsilon\frac{\partial^2\vec{H}}{\partial t^2}$$

... wave eqns in $\vec{H}$

Comments :

① Hence has the e.m. waves propagates in general meadium it is subjective to attenuation as well as phase change.

Therefor the elec. field strength decrease as the wave propagate in a particular direction which is given by poynting vector.

(2) The behaviour of elec. & mag. field are excatly same except that the ele. & mag. fields are prependuler to each other.

Special case :

---- for sinusoidally varying fields.

$$\frac{\partial}{\partial t} \rightarrow j\omega$$

$$\frac{\partial^2}{\partial t^2} \rightarrow -\omega^2$$

$$\therefore \quad \nabla^2 \cdot \vec{E} = \mu\sigma \cdot j\omega\vec{E} + \mu\epsilon(-\omega^2\vec{E})$$

$$\longrightarrow \boxed{\nabla^2\vec{E} = \gamma^2\vec{E}}$$

$$\gamma^2 = \mu\sigma\, j\omega - \omega^2\mu\epsilon$$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

$$= \alpha + j\beta$$

if $\sigma = 0$

---- lossless Meadium.

$$\gamma = \sqrt{j\omega\mu(0 + j\omega\epsilon)}$$

$$= \boxed{j\omega\sqrt{\mu\epsilon} = \alpha + j\beta}$$

$$\alpha = 0$$

$$\beta = \omega\sqrt{\mu\epsilon} = \frac{\omega}{v_p}$$

$$\boxed{v_p = \frac{1}{\sqrt{\mu\epsilon}}}$$

if $\mu = \mu_0$, $\epsilon = \epsilon_0$ } --- for free space

$$v_p = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = c = 3\times10^8\ m/sec.$$

60

$$\gamma = 0 + j\beta$$

$$\gamma^2 = -\beta^2$$

$$\boxed{\nabla^2 \vec{E} = -\beta^2 \vec{E}}$$ --- wave equ for lossless Meadium $(\sigma = 0)$

Ex: An em wave is propagating in a general lossy meadium has been const of μ, ϵ & σ in +z direction and has only x-components of electrical field.

assuming sinusoidal variation find the value the elect. field components.

prop. --- along +z

$$\vec{E} = E_x \hat{a}_x$$

Meadium : μ, ϵ, σ

Sinusoidal Variation

To find

E_x

$$\nabla^2 \vec{E} = \gamma^2 \vec{E}$$

$$(\nabla^2 E_x)\hat{a}_x + (\nabla^2 \cancel{E_y})\hat{a}_y + (\nabla^2 \cancel{E_z})\hat{a}_z = \gamma^2 (E_x \hat{a}_x + \cancel{E_y}\hat{a}_y + \cancel{E_z}\hat{a}_z)$$

$$\nabla^2 E_x = \gamma^2 E_x$$

$$\Rightarrow \cancel{\frac{\partial^2 E_x}{\partial x^2}} + \cancel{\frac{\partial^2 E_x}{\partial y^2}} + \underbrace{\frac{\partial^2 E_x}{\partial z^2} = \gamma^2 E_x}$$

characteristic eqn $m^2 = \gamma^2$

$m = \pm\gamma$

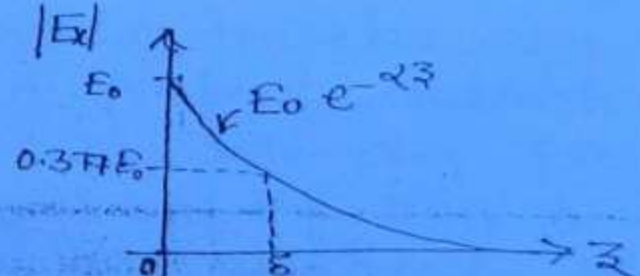
$$E_x = A e^{+\gamma z} + \underbrace{B\cdot e^{-\gamma z}}_{\text{along } +z}$$

$$\boxed{E_x = E_0 e^{-\gamma z}}$$ Ans.

Depth of penetration :

$$E_x = E_0 e^{-\gamma z}$$
$$= E_0 e^{-(\alpha + j\beta) z}$$
$$= E_0 e^{-\alpha z} \cdot e^{-j\beta z}$$

$$\boxed{|E_x| = E_0 \cdot e^{-\alpha z}}$$



if $\alpha z = 1$ for $z = \delta$ ..... depth of penetration

$\alpha . \delta = 1$

$$\boxed{\delta = \frac{1}{\alpha}}$$

$E_x = E_0 \cdot e^{-\alpha z} = E_0 e^{-1}$ at $z = \delta$

$\cong 0.377\, E_0$

① As the wave enter in a lossy Meadium having finite conductivity in elect. field strength decrease exponentially.

② the depth of peneration or the skin depth represents total distant trawel by em.waves where its elect field strength decrease to 37% of it's initial value.

③ The depth of peneration is inversly equal to the attenuation const. α.

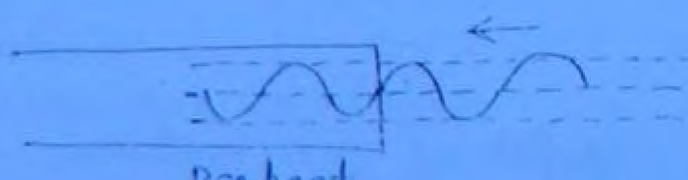
④ Higher is the conductivity of meadium, Higher is the value of attenuation const α, & therefore lower is the value of depth of peneration & vice-versa.

Case I :- Perfect diel. ($\sigma = 0$)

$$\gamma = \alpha + j\beta$$
$$= \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

$$\Rightarrow \alpha = 0$$

$$\boxed{\delta = \infty} = \frac{1}{\alpha}$$



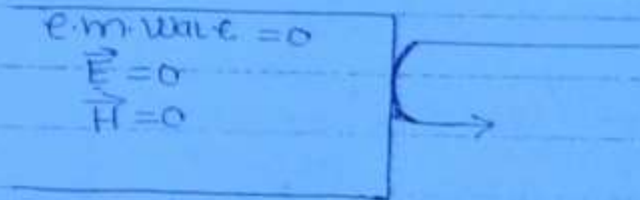
perfect diel.

Case : 2. perfect conductor

$(\sigma \to \infty)$

$\Rightarrow \alpha \to \infty$

$\delta = \frac{1}{\alpha} \to 0$



perfect cond.

$(\sigma \to \infty)$

(1) for a perfect diel. $\sigma = 0$, $\alpha = 0$ therefor depth of penetration is infinite.

In such meadium the wave travel without any attenuation & therefor elel. field strength remains const at all point.

(2) for a perfect cond. the depth of penetration is zero, the wave cannot enter in such meadium therefor any perfect conductor behaves as a perfect reflector.

(3) In side any perfect cond. emw, elcl. field and the mag. field do not exists.

(4) any perfect conductor behaves as an electromangetic mirror.

ε for a good conductor:

$$\boxed{\frac{\sigma}{\omega\epsilon} >> 1}$$

$$\delta = \frac{1}{2}.$$

64

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

$$= \sqrt{j\omega\mu\sigma\left(1 + \cancel{\frac{j\omega\epsilon}{\sigma}}\right)}$$

$j = \sqrt{-1}$

$\sqrt{j} = \sqrt{\sqrt{-1}}$

complicated so phase angle taken

$$\cong \sqrt{\omega\mu\sigma}\ \angle 45^\circ$$

$$= \sqrt{\omega\mu\sigma}\left(\underbrace{\cos 45^\circ}_{=1/\sqrt{2}} + j\underbrace{\sin 45^\circ}_{1/\sqrt{2}}\right)$$

$$= \sqrt{\frac{\omega\mu\sigma}{2}}(1+j) = \alpha + j\beta$$

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}}$$

$$\boxed{\delta = \frac{1}{\sqrt{\pi f \mu\sigma}}}$$

$$\delta \propto \frac{1}{\sqrt{f}}$$

$f \uparrow \rightarrow \delta \downarrow$

$f \downarrow \rightarrow \delta \uparrow$

Comments :- for a specified material if the freq of operation is High then the depth of pene. is High & there for thin conductor are preferable.

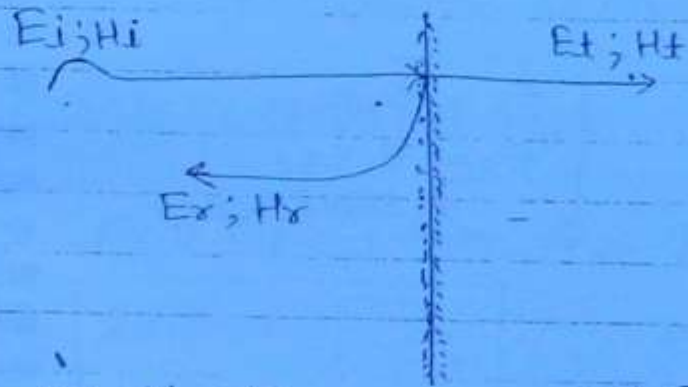
at low freq. since the depth of penetration is High we have to use thick conductor

so that entire powr is combined with in the conducting region.

65

☆ Reflection & Refraction of EMW

---- normal incidence



$$\left\{ \begin{aligned} \frac{E_i}{H_i} &= \eta_1 \\ \frac{E_r}{H_r} &= -\eta_1 \\ \frac{E_t}{H_t} &= \eta_2 \end{aligned} \right.$$

Meadium ①: $\mu_1, \epsilon_1, \sigma_1$, η_1

med. ②: $\mu_2, \epsilon_2, \sigma_2$, η_2

$$\frac{Z_L - Z_0}{Z_L + Z_0} \Big/ \frac{2Z_L}{Z_L + Z_0}$$

$$\boxed{\left(\frac{E_r}{E_i}\right) = \rho = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}}$$

$$\left(\frac{E_t}{E_i}\right) = T = \frac{2\eta_2}{\eta_2 + \eta_1}$$

$$\left(\frac{H_r}{H_i}\right) = \rho' = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2} = -\rho$$

$$\left(\frac{H_t}{H_i}\right) = T' = \frac{2\eta_1}{\eta_1 + \eta_2}$$

perfect diel.

$$\sigma = 0$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} \cong \sqrt{\frac{\mu_0}{\epsilon}}$$

$$\eta_1 \propto \sqrt{\frac{1}{\epsilon_1}}$$

$$\eta_2 \propto \sqrt{\frac{1}{\epsilon_2}}$$

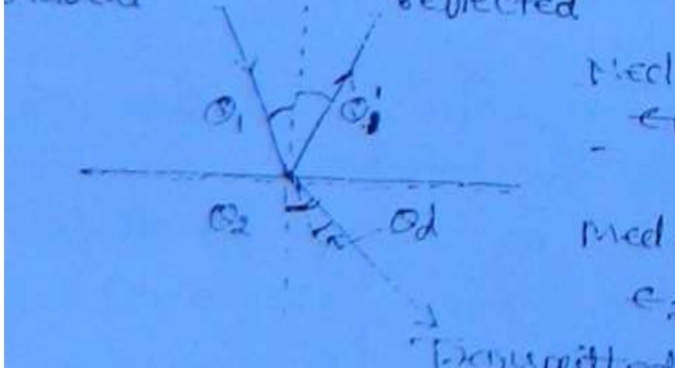
$$\boxed{\begin{aligned} \rho &= \frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}} \\ T &= \frac{2\sqrt{\epsilon_1}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}} \\ \rho' &= \frac{\sqrt{\epsilon_2} - \sqrt{\epsilon_1}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}} \\ T' &= \frac{2\sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}} \end{aligned}}$$

$$\left\{ \begin{aligned} \epsilon_1 &= \epsilon_0 \epsilon_{r1} \\ \epsilon_2 &= \epsilon_0 \epsilon_{r2} \end{aligned} \right.$$

is same result

---- for perfect diel.

Oblique incidence :-



Med. ①

$\epsilon_1 ; (\sigma_1 = 0)$

Med. ②

$\epsilon_2 ; (\sigma_2 = 0)$

θ_d = angle of deviation

$\theta_d = \theta_2 \sim \theta_1$

$\theta_1' = \theta_1$ ---- law of reflection

$$\boxed{\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1} = \sqrt{\frac{\epsilon_1}{\epsilon_2}}}$$

---- Snell's law

$$v_1 = \frac{1}{\sqrt{\mu_0 \epsilon_1}}$$

$$v_2 = \frac{1}{\sqrt{\mu_0 \epsilon_2}}$$

$$\frac{v_2}{v_1} = \sqrt{\frac{\epsilon_1}{\epsilon_2}}$$

Case 1: Critical angle

$\theta_c = \theta_1$ when $\theta_2 = 90°$

$$\boxed{\sin\theta_c = \sqrt{\frac{\epsilon_2}{\epsilon_1}}} \; ; \quad \epsilon_2 < \epsilon_1$$

$\theta_1 \geq \theta_c$ --- total internal reflection (TIR) $\left.\right\}$ $\boxed{\begin{matrix} \rho = 1 \\ T = 0 \end{matrix}}$

Case 2: Brewster's angle

$\theta_1 = \theta_B$

$$\boxed{\tan\theta_B = \sqrt{\frac{\epsilon_2}{\epsilon_1}}} \; ; \quad \boxed{\begin{matrix} \rho = 0 \\ T = 1 \end{matrix}}$$

① The critical Angle represent the angle of incident for which the angle of transmisson is 90°.

② total internal reflection occures whenever the angle of incident is more than the critical Angle.

this phenomenon is impot. for the propagation of waves in boundary medium such as propagation through

optical fibre.

68

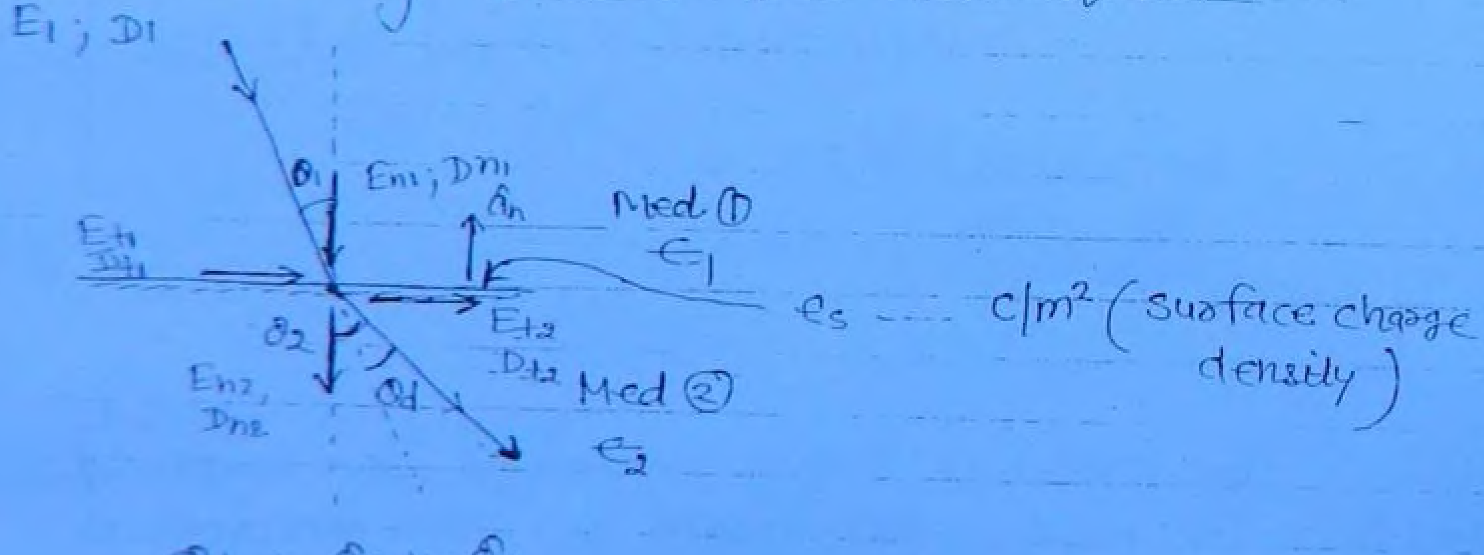
The Brewster's angle represent the angle of incident for which the wave is not reflected & the entire wave is transmitted in IInd Medium.

this phenomenon is

for the propagation for e.m.waves in unbound medium.

Such as the propagation through free space. or the transmission of waves through antenna.

Boundary Relation for elect. fields –



$\theta_d = \theta_2 \sim \theta_1$

Tatind :-

$E_{t1} \longleftrightarrow E_{t2}$ ..... tangential comp. relation.

$E_{n1} \longleftrightarrow E_{n2}$ ---- normal Compt. relation.

$\left.\begin{matrix}\theta_1 \\ \theta_2\end{matrix}\right\} \longleftrightarrow \left\{\begin{matrix}\epsilon_1 \\ \epsilon_2\end{matrix}\right.$ ---- Oblique incidence of electric fields.

Tangential compts. Relations :-

$$\boxed{\begin{gathered} E_{t1} = E_{t2} \\ \frac{D_{t1}}{\epsilon_1} = \frac{D_{t2}}{\epsilon_2} \\ \hat{a}_n \times (\vec{E_1} - \vec{E_2}) = 0 \end{gathered}}$$

$|\hat{a}_n \times \vec{E_1}| = E_{t1}$

$|\hat{a}_n \times \vec{E_2}| = E_{t2}$

~~$|\hat{a}_n \times \vec{E_2}| = E_{t2}$~~

$\hat{a}_n$ .... unit normal vector $\perp$ to common boundary.

Case 1 : Med. (1) is conduct.

$E_{t1} = 0$

$E_{t1} = E_{t2} = 0$

(1) The tangential compts of elect. field intensity E is continous across a common boundary separating two diff. die. meadium.

(2) the result is independent of surface charge density present on the common boundary.

(3) The tangential compts. of elet. field intensi E will not exist across a common boundary separating two diff. media when one of them or both media are conducting media.

normal comp. relation (70)

$$\rightarrow \boxed{\begin{array}{l} D_{n1} - D_{n2} = \rho_s \\ \epsilon_1 E_{n1} - \epsilon_2 E_{n2} = \rho_s \\ \hat{a}_n \cdot (\vec{D_1} - \vec{D_2}) = \rho_s \end{array}}$$

$\hat{a}_n \cdot \vec{D_1} = D_{n1}$

$\hat{a}_n \cdot \vec{D_2} = D_{n2}$

Case : 1 $\quad \rho_s = 0$

$$\boxed{D_{n1} = D_{n2}}$$

Case : 2 $\quad$ Med (2) is cond.

$$D_{n2} = 0$$

$$D_{n1} - \cancel{D_{n2}} = \rho_s$$

$$\boxed{D_{n1} = \rho_s}$$

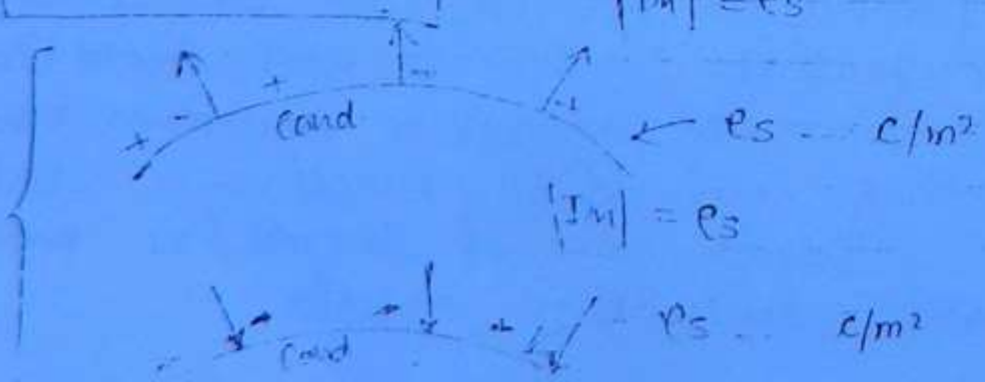
Case 3 :- $\quad$ Med (1) is cond.

$$\cancel{D_{n1}} - D_{n2} = \rho_s$$

$$\boxed{-D_{n2} = \rho_s}$$

from case (2) & (3)

$$\boxed{|D_n| = \rho_s}$$



① The normal comp. of elect. flux density D is discontinous by a facotor ρ_s. where ρ_s repst. surface charge density on the common boundary separating two diff. dielet. Media.

② for a charge free common boundary the normal comp of elel. flux density D is alway continous.

③ Any conducting surface always supports normal comp. of elect. field such that the normal comp. of elect. flux density D is numerically equal to Magnitude of surface charge density present on the conducting surface.

Oblique Incidence

if $\rho_s = 0$

$$\boxed{\begin{matrix} E_{t1} = E_{t2} \\ D_{n1} = D_{n2} \end{matrix}}$$

$$\begin{cases} E_{t1} = E_1 \cdot \sin\theta_1 \\ E_{t2} = E_2 \sin\theta_2 \end{cases}$$

$$\begin{cases} E_{n1} = E_1 \cdot \cos\theta_1 \\ E_{n2} = E_2 \cos\theta_2 \end{cases}$$

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_1}{\epsilon_2}} = \frac{\epsilon_{r1}}{\epsilon_{r2}}$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

---- Snell's law

Ex:

$\hat{a}_n = +\hat{a}_z$

Med ① ; $\boxed{z > 0}$

$\epsilon_1 = 4\epsilon_0$

$\vec{E_1} = 3\hat{a}_x - 6\hat{a}_y - 8\hat{a}_z$

$\rho_s = 3\epsilon_0$ -- C/m^2

$\vec{E_2}$

Med ② ; $\boxed{z < 0}$

$\epsilon_2 = 6\epsilon_0$

To find

$\vec{E_2} = ?$

$= E_{x2}\hat{a}_x + E_{y2}\hat{a}_y + E_{z2}\hat{a}_z$

$\vec{E_2}$ — tangential comp. $E_{t1} = E_{t2}$ — $x \Rightarrow E_{x2} = E_{x1} = 3$; $y \Rightarrow E_{y2} = E_{y1} = -6$

— normal comp. $D_{n1} - D_{n2} = \rho_s$ } $z \Rightarrow D_{z1} - D_{z2} = \rho_s$

$\epsilon_1 E_{z1} - \epsilon_2 E_{z2} = \rho_s$

$= 4\cancel{\epsilon_0}(-8) - 6\cancel{\epsilon_0}E_{z2} = 3\cancel{\epsilon_0}$

$E_{z2} = \frac{-32-3}{6} = -\frac{35}{6}$

$\vec{E_2} = 3\hat{a}_x - 6\hat{a}_y - \frac{35}{6}\hat{a}_z$

Date

Boundary relations for Mag. fields

tangential comp. relation (23)

$$\boxed{\begin{gathered} H_{t1} - H_{t2} = J_s \\ \frac{B_{t1}}{\mu_1} - \frac{B_{t2}}{\mu_2} = J_s \\ \hat{a}_n \times (\vec{H_1} - \vec{H_2}) = \vec{J_s} \end{gathered}}$$

; J_s – Surface current density on common boundary ... A/m

$$|\hat{a}_n \times \vec{H_1}| = H_{t1}$$

$$|\hat{a}_n \times \vec{H_2}| = H_{t2}$$

$H_{t1} = H_{t2}$ --- $J_s = 0$

$H_{t1} = J_s$ --- if Med. (2) is cond.

$-H_{t2} = J_s$ --- if Med (1) is cond.

$$|H_t| = J_s$$

(1) The tangential comp of Mag. field intensity H is a discontinuous by a factor of J_s. J_s represent the surface current density present on the common boundary separating two diff. boundary media.

(2) On a ~~current~~ current free common boundary the tangential comp of the H field is always continuous.

(3) Any conducting surface will support only the tangential comp of H field such that the magnitude of mag. field intensity H is numerically equal to the surface current density present on the conducting surface

Normal Comp. relation

(74)

$$\boxed{\begin{aligned} B_{n1} &= B_{n2} \\ \mu_1 H_{n1} &= \mu_2 H_{n2} \\ \hat{a}_n \cdot (\vec{B_1} - \vec{B_2}) &= 0 \end{aligned}}$$

$$\hat{a}_n \cdot \vec{B_1} = B_{n1}$$
$$\hat{a}_n \cdot \vec{B_2} = B_{n2}$$

$B_{n1} = B_{n2} = 0$ if the meadium (1) & med. (2) or both are conducts.

(1) The normal comp of mag. flux density B is continous across a common Boundary separating two different mag. media.

(2) The result is independent of the surface current density present on the common boundary.

(3) The normal comp. of mag. flux density B will not exist across a common Boundary separating two different media. when one of them over both media are conducting media

Oblique Incidence

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2}} = \frac{\mu_{r1}}{\mu_{r2}}$$

Ex.

Med. 1 ; $\boxed{x > 0}$

$\mu_1 = 2\mu_0$

$\vec{B_1} = 4\hat{a}_x + 5\hat{a}_y - 6\hat{a}_z$

$\hat{a}_n = +\hat{a}_x$

$J_s = 0$

$\vec{B_2} = ?$

Med 2 ; $\boxed{x < 0}$

$\mu_2 = 7\mu_0$

$\vec{B_2}$ — normal

$B_{n1} = B_{n2} \longrightarrow B_{x2} = B_{x1} = 4$

$\vec{B_2}$ — tangential

$H_{t1} - H_{t2} = J_s$

$H_{t1} = H_{t2}$

— $H_{y1} = H_{y2} \Rightarrow \frac{B_{y2}}{\mu_2} = \frac{B_{y1}}{\mu_1}$; $B_{y2} = \frac{\mu_2}{\mu_1} \cdot B_{y1}$

$= 7/2 \times 5 = \frac{35}{2}$

— $H_{z2} = H_{z1}$

$\Rightarrow B_{z2} = \frac{\mu_2}{\mu_1} \cdot B_{z1}$

$\Rightarrow \frac{7}{2} \times (-6) = -21$

$\vec{B_2} = 4\hat{a}_x + \frac{35}{2}\hat{a}_y - 21\hat{a}_z$

Wave polarization (76)

(1) The wave polarization is related to the orintation of elet. field vector associated with the EMW.

(2). if the vertical entina installed, the electrical field vector is also vertical & EMW is vertically polarised.

(3) if any antina is Horizontally polari the elel. fied vector is parallel to the surface at the earth & the EMW is Horizontally polarised.

(4) the orintation of the elel field vector at the transmitting & receiving end must be same so that max. induced emf. is obtained at the receiving antina.

therefor the polarization the transmitting & Receiving antina must be identical.

$$E_x = E_1 \sin(\omega t - \beta z)$$

$$E_y = E_2 \sin(\omega t - \beta z + \alpha)$$

time phase angle b/w E_x & E_y

To findout

effect of $\underbrace{E_1, E_2; \alpha}$

Where $z = 0$ plane

$$E_x = E_1 \sin \omega t$$

$$E_y = E_2 \sin(\omega t + \alpha)$$

$\alpha = 0°$ .... In same phase.
$\alpha = 90°$ .... quadrature phase.

Case 1 :- linear polarization

$$\boxed{\begin{array}{c} E_1 \neq E_2 \\ \alpha = 0° \text{ or } 180° \end{array}}$$

if $\alpha = 0°$

$E_x = E_1 \sin\omega t$
$E_y = E_2 \sin\omega t$

$$\frac{E_y}{E_x} = \frac{E_2}{E_1} = m$$

$$\boxed{E_y = mE_x}$$ --- eqn of st. line.

(a) if $E_1 = 0$

$$\begin{cases} E_x = 0 \\ E_y = E_2 \sin\omega t \end{cases}$$

--- E.M.W is linearly polarised along y- direction.
---- Wave is Vertically polarized

(b) if $E_2 = 0$

$$\begin{cases} E_x = E_1 \sin\omega t \\ E_y = 0 \end{cases}$$

--- EMW is linearly polarized along the x- direction.
--- Wave is Horizontally polarized.

Case : 2 Circular polarization

$$\boxed{\begin{array}{l} E_1 = E_2 = E_0 \\ \alpha = \pm 90^\circ \end{array}}$$

$E_x = E_0 \sin\omega t$

$E_y = E_0 \sin(\omega t + 90)$; $\alpha = \pm 90^\circ$

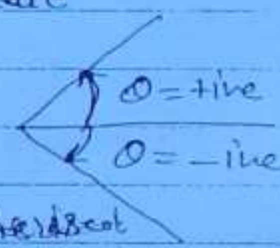
$= E_0 \cos\omega t$

$$\boxed{E_x^2 + E_y^2 = E_0^2}$$ --- eqn of a circle

$$\boxed{\begin{array}{l} \alpha = +90^\circ \\ \alpha = -90^\circ \end{array}}$$

--- LCP wave

--- RCP wave



~~Right circular polarisation~~

~~Left-hand circular polarisation~~

Case 3: Elliptical polarization

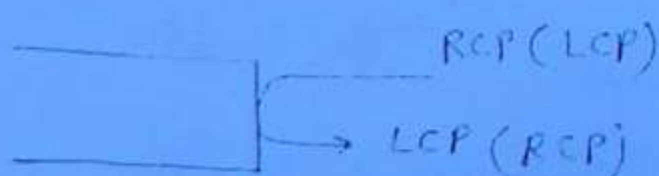
$$\boxed{\begin{array}{l} E_1 \neq E_2 \\ \alpha = \pm 90^\circ \end{array}}$$

$$\boxed{\begin{array}{l} \alpha = +90^\circ \\ \quad = -90^\circ \end{array}}$$

--- LEP wave

--- REP wave

Main points :- ① When RCP wave incidende on a perfect conductor perpendualary, the reflected wave is LCP wave & vice-versa.



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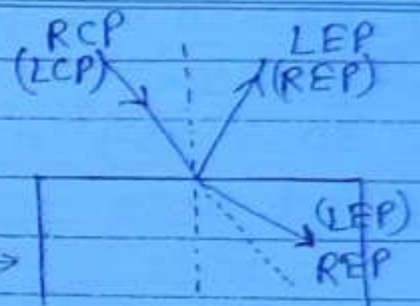
(2) Polystyrene



79

if an RCP wave incidence on polystyrene then

(a) Transmitted wave ~~are~~ is REP wave
(b) Reflected wave is LEP wave & vice-vers

(3) if is a circularly polarizied wave or elepticall polarizied wave incidence at $\boxed{\text{Brewster's}}$ angle on any interface then the reflected wave & transmitted wave are linearly polarizied.

Ex: $\vec{E} = 5 \sin(\omega t - \beta z)\hat{a}_x + 10 \sin(\omega t - \beta z)\hat{a}_y$ $\Big|_{\alpha = 0}$

$E_1 \neq E_2$

-- linearly polarizied

Ex: $\vec{E} = 5 \sin(\omega t - \beta z)\hat{a}_x + 5 \sin(\omega t - \beta z - 90^\circ)\hat{a}_y$

$$\left.\begin{array}{l} E_1 = E_2 \\ \alpha = -90 \end{array}\right\} \text{-- RCP wave}$$

Ex: $\vec{E} = 5 \sin(\omega t - \beta z - 30^\circ)\hat{a}_x + 10 \sin(\omega t - \beta z + 60^\circ)\hat{a}_y$

$E_1 \neq E_2$

$\alpha = 60 - (-30^\circ) = +90^\circ$

LEP wave

Ex: $(5\sin(\omega t - \beta z - 30^\circ)\,\hat{a}_x + (5\sin(\omega t - \beta z + 30^\circ)\,\hat{a}_y$

$$\begin{cases} E_1 = E_2 = 5 \\ \alpha = 30 - (-30^\circ) = 60^\circ \end{cases}$$

80

--- unpolarizied wave

Imp

example:- An emw. having following x-compts. of elel. field is travelling in a lossless medium with $\mu_r = 1$ + $\epsilon_r = 9$.

find:

(1) All the parameters associated with emw.

(2) the Mag. field intensity associated with em.w.

$$\vec{E} = 10 \cdot \cos(6\pi \times 10^3 t - \beta z)\,\hat{a}_x$$

$\sigma = 0$

$\mu_r = 1$

$\epsilon_r = 9$

To find: 1. Various parameters
2. $\vec{H}$

$$\vec{E} = E_x \cdot \hat{a}_x$$

$$E_x = E_m \cdot \cos(\omega t \underbrace{- \beta z}_{+z \text{ direction}})$$

(1) $\vec{E} = E_x \hat{a}_x$

(2) $E_m = 10$ V/m

(3) Wave is linearly polarized along x-direction

(4) Direction of propagation along +z direction.

(5) Intrinsic impedance $\eta = \sqrt{\frac{\mu}{\epsilon}}$

$$\eta = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}} = \frac{\eta_0}{\sqrt{\epsilon_r}} = \frac{120\pi}{3} = 40\pi \; \Omega$$

(6) $$v_p = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_0\mu_r \cdot \epsilon_0 \epsilon_r}} = \frac{c}{\sqrt{\epsilon_r}} = \frac{3\times10^8}{3} = 10^8 \text{ -- m/sec.}$$

(7) $$\omega = 6\pi \times 10^8 = 600\pi \text{ ---- Mrad/sec.}$$

$$f = \frac{\omega}{2\pi} = 300 \text{ -- MHz.}$$

(8) $$\beta = \frac{\omega}{v_p} = \frac{6\pi\times10^8}{10^8} = 6\pi \text{ -- rad/m}$$

(9) $$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{6\pi} = \frac{1}{3} \text{ -- m}$$

(10) $$\vec{P} = \vec{E}\times\vec{H}$$

$$= \frac{1}{2}\eta H_m^2$$

$$= \frac{1}{2}\frac{E_m^2}{\eta}\cdot \hat{a}_z = \frac{1}{2}\times\frac{100}{40}\,\hat{a}_z \text{ --- W/m}^2$$

(11) $$\eta = \frac{E_m}{H_m}\;;\quad H_m = \frac{E_m}{\eta} = \frac{10}{40\pi} = \frac{1}{4\pi} \text{ -- A/m}$$

$$\vec{P} = \vec{E}\times\vec{H}$$

$$\hat{a}_z = \hat{a}_x \times (+\hat{a}_y)$$

$$\vec{H} = H_y \hat{a}_y$$

$$\vec{H} = H_m \cos(\omega t - \beta z)\hat{a}_y$$

$$\frac{1}{4\pi}\cos(6\pi\times10^8 \cdot t - 6\pi z)\hat{a}_y \text{ --- A/m}$$

Ans.

Ex: $\vec{H} = 10\sin(6\pi \times 10^8 t + 6\pi x)\,\hat{a}_z$

Note → Wave is linearly polarized along y-direction.

$\vec{H} = H_z \cdot \hat{a}_z$

Propg. --- $-x$ direction

82

$\vec{P} = -P_x \hat{a}_x$

$\vec{P} = \vec{E} \times \vec{H}$

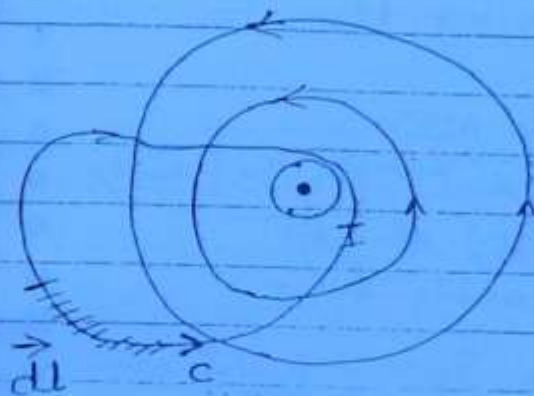
$-\hat{a}_x = (-\hat{a}_y) \times (\hat{a}_z)$

$\vec{E} = -E_y \hat{a}_y$

---- Static Magnetic fields.

$\vec{B}, \vec{H} \neq f(t)$

Ampere's circuital law :-



$\vec{H} ; \vec{B} ; \propto \frac{1}{r^2}$

Find : $\vec{H}$

: $d\vec{l}$

: $\vec{H} \cdot d\vec{l}$

$$\boxed{\oint_c \vec{H} \cdot d\vec{l} = I}$$

(current enclosed)

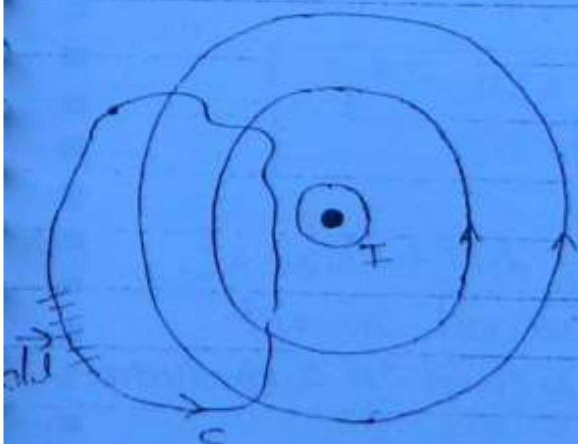
---- Ampere's circuital law in integral form

Where

$I = I$ --- line current

$= \int_l J_s \cdot dl$ --- surface current

$= \iint_s \vec{J} \cdot d\vec{S}$ --- volume current.



$\vec{H} ; \vec{B}$

$$\boxed{\oint_c \vec{H} \cdot d\vec{l} = 0}$$

★ Ampere's circuital law in differential or point form :-

$$\nabla \times \vec{H} = \vec{J_c} + \vec{J_d} = \sigma \vec{E} + \cancel{\frac{\partial \vec{D}}{\partial t}}$$

(84)

$$\boxed{\nabla \times \vec{H} = \vec{J_c} = \sigma \vec{E}.}$$

Statements :-

① The circulation of static mag. field intensity $\vec{H}$ along any closed curve (C) equal to its total current I enclosed with the enclosed ~~current~~ C.

② if no current is enclosed the circulation of $\vec{H}$ field along the closed curve is zero all through at each point. $\vec{H}$ & $d\vec{l}$ has finite value.

③ The law is applicable irrespective of the shape of the closed curve C.

④ The curl of static mag field intensity $\vec{H}$ at any point is always equal to conduction current density present at that point.

Ex:

due to flow of current in opp direction in 2 conductor.

To find :- $\oint_C \vec{H} \cdot d\vec{l} = ?$

$= 3I - (-2I) = 5I$

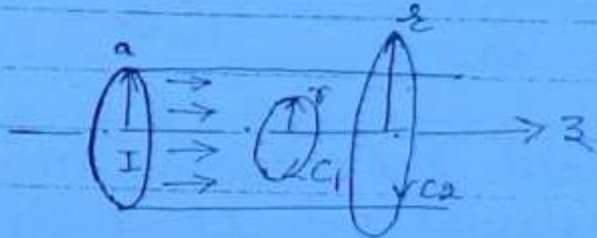
due to opp direction of circulation of current

Date

ex: A long solid cylindrical cylinder radius 'a' carries a uniform current 'I' throughout of cross-section of conductor.

Find the value of mag. flux density $\vec{B}$ at all points/regions.

(85)



to find: $\vec{B}$ for

1. $r < a$
2. $r > a$
3. $r = a$

(1). $r < a$

$$\oint_{c_1} \vec{H} \cdot d\vec{l} = I'$$

$$\frac{I'}{I} = \frac{\pi r^2}{\pi a^2}$$

$$I' = \frac{r^2 \cdot I}{a^2}$$

$$H_\phi \cdot 2\pi \cancel{r} = \frac{r^{\cancel{2}} \cdot I}{a^2}$$

$$H_\phi = \frac{I r}{2\pi a^2} \quad ; \quad \vec{H} = \frac{I r}{2\pi a^2} \hat{a}_\phi$$

$$\boxed{\vec{B} = \frac{\mu I r}{2\pi a^2} \hat{a}_\phi}$$

(2) $r > a$

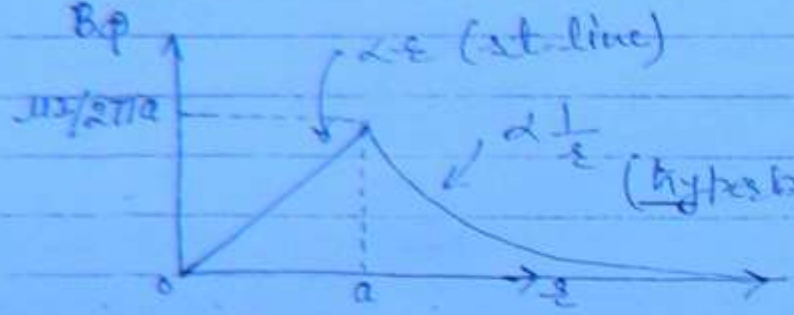
$$\oint_{c_2} \vec{H} \cdot d\vec{l} = I$$

$$H_\phi \cdot 2\pi r = I \quad ; \quad \boxed{\vec{B} = \frac{\mu I}{2\pi r} \hat{a}_\phi}$$

$$\vec{B} = \frac{\mu I r}{2\pi a^2} \hat{a}_\phi \quad \cdot \quad r < a$$

$$= \frac{\mu I}{2\pi r} \hat{a}_\phi \quad \cdot \quad r > a$$

$$\vec{B} = \frac{\mu I}{2\pi a}\hat{a}_\phi \quad \text{---} \quad r = a$$



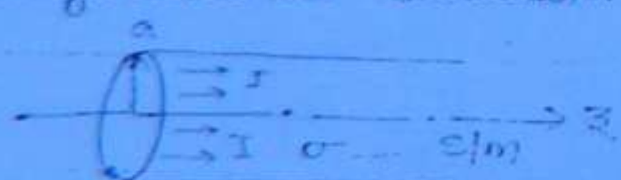
Imp points :- (1) The mag. field increases linearly in a solid cylindrical conductor whereas it decreases in the hyperbolic outside the cylindrical conductor.

(2) The mag. field is always constant on the surface of any conductor.

(3) any cylinderical conductor behaves as an infinite line current since the mag. field due to both configuration is same.

Ex: A long solid cylinderical cond. of radius 'a' & conductivity 'σ' carries uniform current 'I'.

find the poynting vector on the surface of cylinderical conductor.



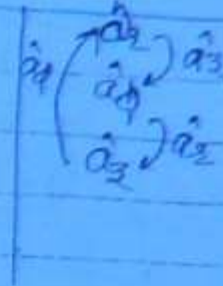
$\vec{P}|_{r=a} = ? = \vec{E} \times \vec{H}$

$\vec{H} = \frac{I}{2\pi a}\hat{a}_\phi$

$\vec{J}_c = \vec{J} = \sigma\vec{E}$

Date

$$\vec{E} = \frac{\vec{J}}{\sigma} = \frac{1}{\sigma}\underbrace{\left(\frac{I}{\pi a^2}\right)}_{\vec{J}\ \ldots\ A/m^2}\hat{a}_z$$



$$\vec{P} = \frac{I}{\pi\sigma a^2}\hat{a}_z \times \frac{I}{2\pi a}\hat{a}_\phi$$

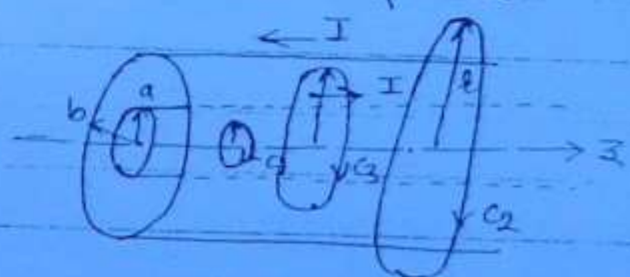
$$= \frac{I^2}{2\pi^2\sigma a^3}\underbrace{(\hat{a}_z \times \hat{a}_\phi)}_{=-\hat{a}_r} =$$

87

$$\vec{P} = \frac{-I^2\,\hat{a}_r}{2\pi^2\sigma a^3}$$

Ex: A long co-axial TL of radia 'a' & 'b' with $b > a$ carries uniform current $\pm I$ on the surface of the 2-cylinderical conductor of the TL.

Calculate mag. field intensity $\vec{H}$ at all points.



To find:

$\vec{H}$ for

1. $r < a$
2. $a < r$
3. $r > b$

(1) $r < a$

$$\oint_{C_1}\vec{H}\cdot d\vec{l} = 0$$

$$\Rightarrow \boxed{H = 0}$$

(2) $r > b$

$$\oint_{C_2}\vec{H}\cdot d\vec{l} = +I + (-I) = 0$$

$$\boxed{\vec{H} = 0}$$

(3) $a < r < b$

$$\underbrace{\oint_{C_3} \vec{H} \cdot \vec{dl}} = I$$

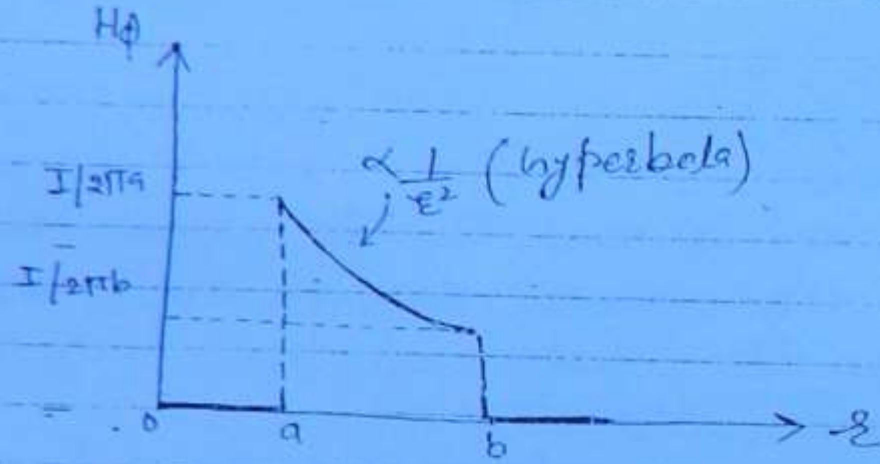
$$H_\phi \cdot 2\pi r = I$$

$$\boxed{\vec{H} = \frac{I}{2\pi r} \hat{a}_\phi}$$

$$\vec{H} = 0 \quad \text{---- } r < a$$

$$= \frac{I}{2\pi r} \hat{a}_\phi \quad \text{--- } a < r < b$$

$$= 0 \quad \text{--- } r > b$$



Magnetic energy density

Date

w_m .... (J/m^3)

$$w_m = \lim_{\Delta v = 0} \left(\frac{\Delta W_m}{\Delta v} \right)$$

$$\boxed{\begin{aligned} w_m &= \frac{1}{2} \mu H^2 \\ &= \frac{1}{2} \vec{B} \cdot \vec{H} \end{aligned}} \quad --(J/m^3)$$

① The mag. energy density represent the mag. energy store per unit volume & gives Mag. energy star at a point in any electromag. region.

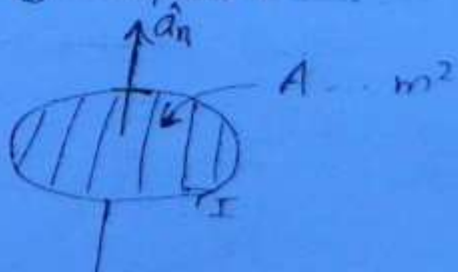
② This depends upon mag. field intensity H & the const of the meadium μ.

Magnetic energy stored

$$\boxed{W_m = \frac{1}{2} L I^2} \quad --- \ J$$

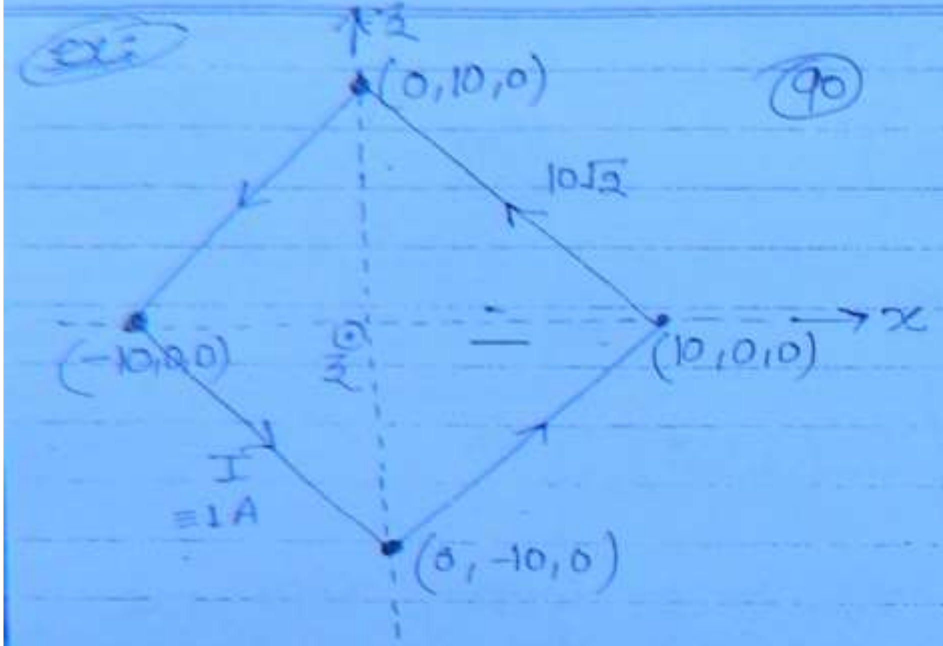
Magnetic diepole moment

Mag. dipole:



$$\boxed{\vec{m}_m = I A \cdot \hat{a}_n} \quad A \cdot m^2$$

Ex:

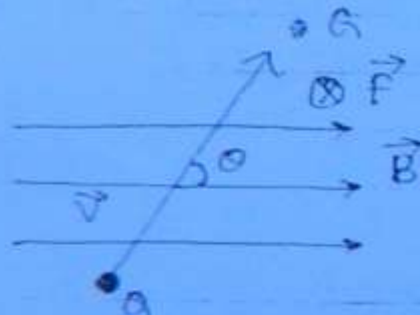


To find :-

$$\vec{m_m} = I A \hat{a}_z$$
$$= 1 \times (10\sqrt{2})^2 \hat{a}_z$$
$$= 200 \hat{a}_z$$
A/m^2

Important main points :-

(1) Moving charge in $\vec{B}$:-



$$\boxed{\vec{F} = Q\vec{v} \times \vec{B}} = Qv B \sin\theta . \hat{a}_n$$

$|F| = 0 \quad ; \quad \theta = 0°$
$= QvB \quad ; \quad \theta = 90°$

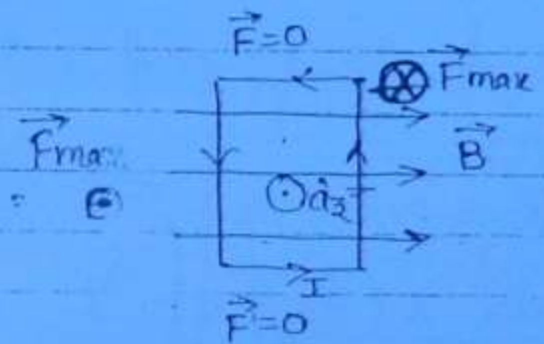
(2) current carrying cond. in $\vec{B}$:-

$$\boxed{\vec{F} = \int_c (\vec{I} \times \vec{B})\, dl}$$

$$\boxed{\left(\frac{\vec{F}}{l}\right) = \vec{I} \times \vec{B}} \text{ — N/m}$$

-- force per unit length

③ current Carrying loop in $\vec{B}$:-

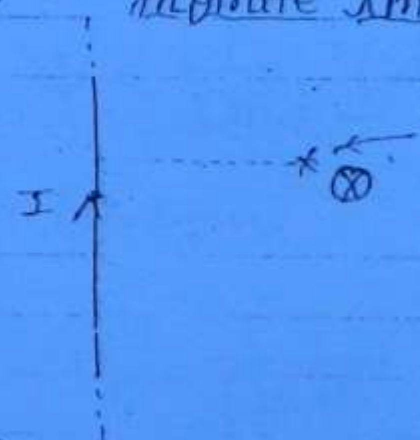


$$\boxed{\vec{T} = \vec{m} \times \vec{B}}$$

$\vec{m} = IA\hat{a}_z$

--- Torque is applicable.

④ Infinite line current :-

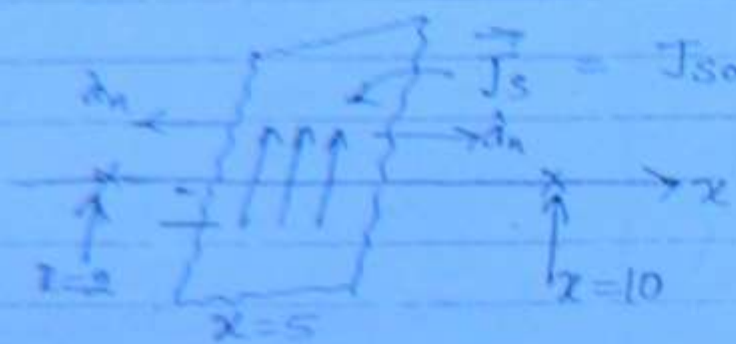


$$B = \frac{\mu I}{2\pi r}$$

⑤ circular loop :-

$$B = \frac{\mu I}{2r}$$

⑥ $\vec{H}$ or $\vec{B}$ due to infinite current sheet:



(92)

$$\boxed{\vec{H} = \frac{1}{2}\vec{J_s}\,\hat{a}_n}$$

$\hat{a}_n \rightarrow$ unit normal vector $\perp$ to current sheet.

at $x=10$

$\hat{a}_n = +\hat{a}_x$

$$\vec{H} = \frac{1}{2}\vec{J_s} \times \hat{a}_n$$

$$= \frac{1}{2} J_{so}\,\hat{a}_z \times (\hat{a}_x)$$

$$= \frac{1}{2} J_{so}\,\hat{a}_y$$

$\hat{a}_x$

$\hat{a}_y$

$\hat{a}_z$

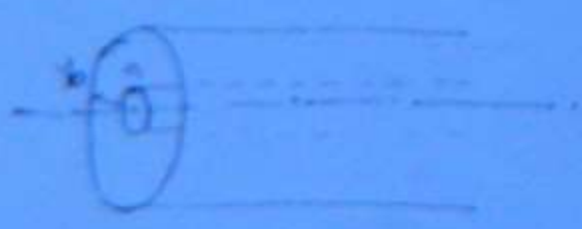
at $x=2$

$\hat{a}_n = -\hat{a}_x$

$$\vec{H} = \frac{1}{2}\vec{J_s} \times \hat{a}_n = \frac{1}{2} J_{so}\,\underbrace{\hat{a}_z \times (-\hat{a}_x)}$$

$$\vec{H} = -\frac{1}{2} J_{so}\,\hat{a}_y$$

⑦ Co-axial T.L. :-



Date

$$\left(\frac{L}{d}\right) \doteq \boxed{\begin{array}{l} L' = \frac{\mu}{2\pi} \ln\left(\frac{b}{a}\right) \\ C' = \frac{2\pi \epsilon}{\ln(b/a)} \end{array}}$$

H/m

F/m

93

$$\boxed{Z_0 = \sqrt{\frac{L'}{C'}} = \frac{1}{2\pi} \sqrt{\frac{\mu}{\epsilon}} \ln\left(\frac{b}{a}\right)} \; ; \Omega$$

$$Z_0 = \frac{\eta}{2\pi} \ln\left(\frac{b}{a}\right)$$

for free space :-

$$\eta = 120\pi$$

$$\boxed{Z_0 = 60 \ln\left(\frac{b}{a}\right)} \; \Omega$$

$$V_p = \frac{1}{\sqrt{L'C'}}$$

$$V_p = \frac{1}{\sqrt{\mu \epsilon}}$$

for free space :

$$\mu = \mu_0 ; \; \epsilon = \epsilon_0$$

$$V_p = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = c$$

Vector Mag. potential ($\vec{A}$)

$\underbrace{\nabla \cdot \vec{B}}_{\text{vector}} = 0$ —— (1)

---- always valid.

(94)

$\nabla \cdot \underbrace{(\nabla \times \vec{A})}_{\text{vector}} = 0$ —— (2)

--- always valid.

---- vector identity.

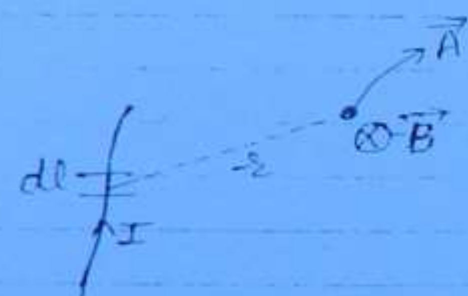
$$\boxed{\vec{B} = \nabla \times \vec{A}}$$ ---- relation b/w $\vec{A}$ & $\vec{B}$

$\vec{B} \perp \vec{A}$

Mathematical defination of $\vec{A}$:-

$$\boxed{\vec{A} = \frac{\mu}{4\pi}\int_c \frac{\vec{I}}{r}\, dl}$$

$\Rightarrow \quad \vec{A} \parallel \vec{I}$



Direction-wise

$\vec{A} \parallel \vec{I}$ $\vec{B} \parallel \vec{I}$
$\vec{B} \perp \vec{A}$ $\Rightarrow$ $\vec{B} = \nabla \times \vec{A}$

(1) There is no physical significance of vector mag. potential A since the mag. charges in the isolated form do not exist in nature.

(2) Calculation of vector A simply gives

—

an intermiddate step.

to calculate the value of mag. flux density B. using the relation

$$\vec{B} = \nabla \times \vec{A}.$$ 95

ex:-

$\vec{A} = \underbrace{2x^2y}_{=A_z}\hat{a}_z$

To find

$\vec{B} = \nabla \times \vec{A}$

$A_x = 0$

$A_y = 0$

$A_z = 2x^2y$

$$\vec{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \cancel{A_x} & \cancel{A_y} & \underbrace{A_z}_{2x^2y} \end{vmatrix} = \hat{a}_x \frac{\partial}{\partial y}(2x^2y) + \hat{a}_y\left[-\frac{\partial}{\partial x}(2x^2y)\right] + \hat{a}_z(0)$$

$$= 2x^2\hat{a}_x - 4xy\,\hat{a}_y$$

$\nabla \times \vec{B} = 0$ .... irrotational vector.

$\neq 0$ --- not an " ... "

units of $\vec{A}$

$$\underset{\frac{Wb}{m^2}}{\vec{B}} = \underset{\frac{1}{m}}{\nabla} \times \underbrace{\vec{A}} \Rightarrow Wb/m$$

---- Static electric fields

$\vec{E}, \vec{D} \neq f(t)$

96

Coulomb's law :-

$$\vec{E} = \frac{Q}{4\pi\epsilon r^2}\hat{a}_r$$

$$\vec{r} = r\hat{a}_r$$

$$\hat{a}_r = \frac{\vec{r}}{r}$$

$$\boxed{\vec{E} = \frac{Q\vec{r}}{4\pi\epsilon r^3}}$$

Where

$$\vec{r} = (x'-x)\hat{a}_x + (y'-y)\hat{a}_y + (z'-z)\hat{a}_z$$

$$|\vec{r}| = \{(x'-x)^2 + (y'-y) + (z'-z)\}^{1/2}$$

⇒ A point charge of +3 nano coulomb is placed at a point (2, 1, -2) in a medium whose dielec. const. is 3.

find the elect. field intensity at point (1, 3, -1).

r - source point (2, 1, -2)
r' - field point (1, 3, -1)

$Q = +3nC$

$\epsilon_r = 3$

To find :- $\vec{E} = \frac{Q\vec{r}}{4\pi\epsilon r^3}$, $\epsilon = \epsilon_0\epsilon_r$

$= (1-2)\hat{a}_x + (3-1)\hat{a}_y + (-1-(-2))\hat{a}_z$

Date

$\vec{r} = -\hat{a}_x + 2\hat{a}_y + \hat{a}_z$

$|\vec{r}| = \sqrt{1+4+1} = \sqrt{6}$

97

$$\vec{E} = \frac{3\times10^{-9}(-\hat{a}_x + 2\hat{a}_y + \hat{a}_z)}{4\pi\times\frac{1}{36\pi}\times10^{-9}\times3(\sqrt{6})^3}$$

$$\vec{E} = \frac{27}{3\times 6\sqrt{6}}(-\hat{a}_x + 2\hat{a}_y + \hat{a}_z)$$

$$\vec{E} = K(-\hat{a}_x + 2\hat{a}_y + \hat{a}_z) \text{ ---- V/m}$$

$$K = \frac{3}{2\sqrt{6}}$$

$$|\vec{E}| = K\sqrt{6} = \frac{3}{2} \text{ — V/m}$$

$$\vec{E} = E\hat{a}_1$$

$$\hat{a}_1 = \frac{\vec{E}}{E} = \frac{K}{3/2}(-\hat{a}_x + 2\hat{a}_y + \hat{a}_z)$$

$$\hat{a}_1 = \frac{1}{\sqrt{6}}(-\hat{a}_x + 2\hat{a}_y + \hat{a}_z)$$

$$\alpha = \cos^{-1}\frac{E_x}{E} = \cos^{-1}\left(\frac{1}{\sqrt{6}}\right)$$ w.r.t. x-axis.

$$\beta = \cos^{-1}\frac{E_y}{E} = \cos^{-1}\left(\frac{2}{\sqrt{6}}\right)$$ ---- w.r.t y-axis

$$\gamma = \cos^{-1}\frac{E_z}{E} = \cos^{-1}\left(\frac{1}{\sqrt{6}}\right)$$ --- w.r.t. z-axis.

★ Gauss' law :-

Date

$$\boxed{\oint\!\!\oint_S \vec{D}\cdot d\vec{s} = Q}$$ --- In integral form

$\underbrace{}_{\text{net elel. flux}}$

(98)

where $Q = Q$ ---- Point charge

$= \int_c \rho_L \cdot dl$ ---- line charge

$= \iint_s \rho_s \cdot ds$ ---- surface charge

$= \iiint_v \rho \cdot dv$ --- Volume charge.

$$\boxed{\nabla\cdot\vec{D} = \rho}$$ --- differential form or point form.

$$\nabla\times\vec{E} = -\cancel{\frac{\partial\vec{B}}{\partial t}}$$

$$\boxed{\nabla\times\vec{E} = 0}$$ irrotational static elet field.

$$\boxed{\oint_c \vec{E}\cdot d\vec{l} = 0}$$

* points :-

① The net elel. flux passing through any closed surface area is always equal to total charge enclosed by the closed surface Area S.

The eqn. is valid for any type of the closed surface Area S.

The divergsion of elel. flux density $\vec{D}$

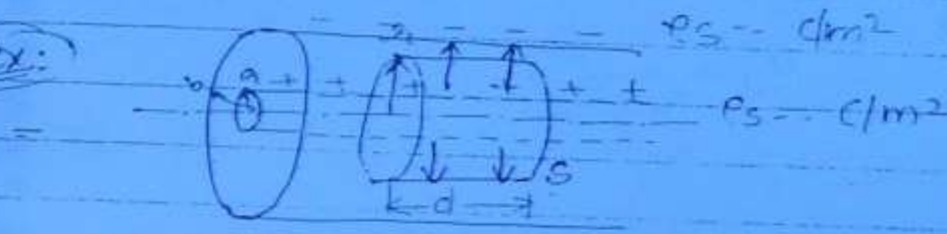
at any point if the elec. mag. Region is always equal to the volume charge density at that point.

99

(4) The curl of static elec. field intensity $\vec{E}$ at any point is always zero & therefor such field is always irrotational.

therefor the circulation of the static elec. field along any closed curve C is always equal to zero.

ex:



To find $\vec{E}$; $\vec{D}$ for $a < r < b$

$$\oiint_S \vec{D}\cdot d\vec{S} = Q = \iint_S \rho_s \cdot ds$$

$$\underbrace{\oiint_S \vec{D}\cdot d\vec{S}}_{\epsilon E_r \cdot 2\pi r d} = Q = \underbrace{\iint_S \rho_s \cdot ds}_{\rho_s \cdot 2\pi r \cdot d \,\big|_{r=a}}$$

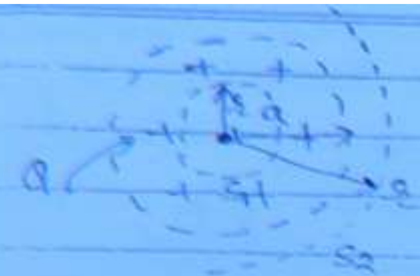
$$E_r = \frac{\rho_s \cdot a}{\epsilon r}$$

$$\boxed{\vec{E} = \frac{\rho_s \cdot a}{\epsilon r}\hat{a}_r}$$

$$\vec{D} = \epsilon \vec{E}$$

$$\boxed{\vec{D} = \frac{\rho_s \cdot a}{r} \cdot \hat{a}_r}$$

ex: A charge +Q is distributed through out the volume of Region of Radius 'a'. find the value of elect filed Intesity $\vec{E}$ at all point.



To find $\vec{E}$ for

1. $r < a$
2. $r > a$
3. $r = a$

① $r < a$

$$\underbrace{\oint\!\!\oint_{S_1} \vec{D}\cdot d\vec{S}}_{\epsilon E_r \cdot 4\pi r^2} = \underbrace{Q'}$$

$$\frac{Q'}{Q} = \frac{4/3\,\pi r^3}{4/3\,\pi a^3}$$

$$Q' = \frac{Q r^3}{a^3}$$

$$E_r = \frac{Q r}{4\pi\epsilon a^3}$$

$$\boxed{\vec{E} = \frac{Q r}{4\pi\epsilon a^3}\,\hat{a}_r}$$

② $r > a$

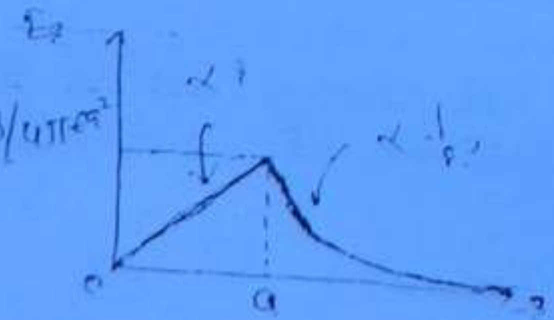
$$\underbrace{\oint\!\!\oint_{S_2} \vec{D}\cdot d\vec{S}}_{\epsilon E_r \cdot 4\pi r^2} = Q$$

$$\boxed{\vec{E} = \frac{Q}{4\pi\epsilon r^2}\,\hat{a}_r}$$

$$\vec{E} = \frac{Q r}{4\pi\epsilon a^3}\,\hat{a}_r \quad \text{--- } r < a$$

$$= \frac{Q}{4\pi\epsilon r^2}\,\hat{a}_r \quad \text{--- } r > a$$

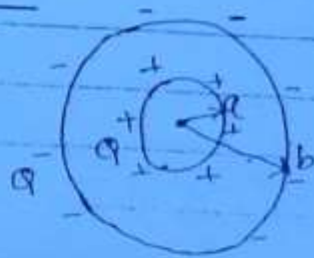
$$= \frac{Q}{4\pi\epsilon a^2}\,\hat{a}_r \quad \text{--- } r = a$$



Date

ex: Two spherical cells of Radia 'a' & 'b'. With b greater than a & equal & opposite charges $\pm Q$ on their surfaces.

find elet field intensity $\vec{E}$ at all points.



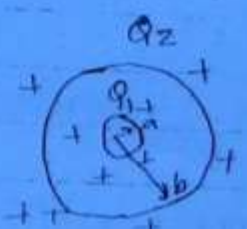
To find:

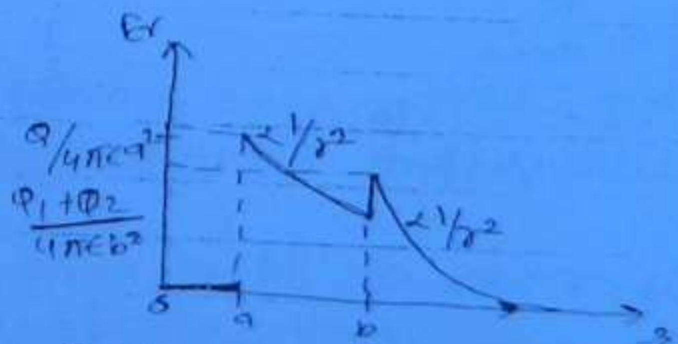
$\vec{E}$ for

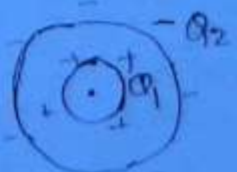
1. $r < a$
2. $a < r < b$
3. $r > b$

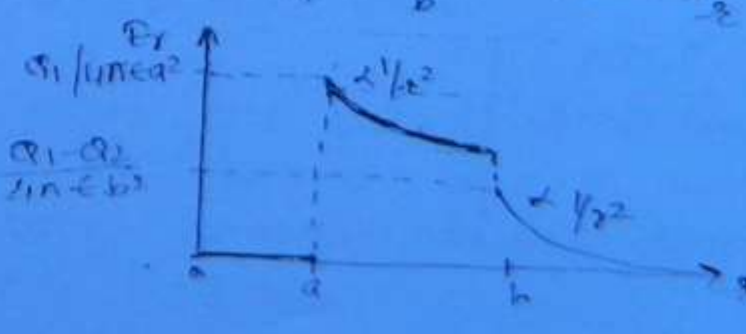
$$\begin{cases} r < a \; ; & \vec{E} = 0 \\ a < r < b \; ; & \vec{E} = \dfrac{Q}{4\pi\epsilon r^2}\, \hat{a}_r \\ r > b \; ; & \vec{E} = 0 \end{cases}$$

practice









Electric energy density

w_e --- J/m^3

$$w_e = \lim_{v \to 0} \left(\frac{\Delta W_e}{\Delta v} \right)$$

102

$$\boxed{w_e = \frac{1}{2} \epsilon E^2 = \frac{1}{2} \vec{D} \cdot \vec{E}}$$ --- J/m^3

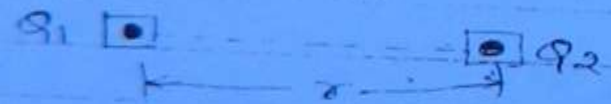
electric energy stord

$$\boxed{W_e = \frac{1}{2} C V^2 = \frac{1}{2} Q V = \frac{1}{2} \frac{Q^2}{C}}$$ --- J

since $Q = C \cdot V$

electric energy stored in a system of 2 charge:

$$W_{e1} = \frac{1}{2} Q_1 V_2 = \frac{1}{2} Q_1 \frac{Q_2}{4\pi\epsilon r} \longrightarrow \frac{1}{2} \frac{Q_1 Q_2}{4\pi\epsilon r}$$



$$W_{e2} = \frac{1}{2} Q_2 V_1 = \frac{1}{2} Q_2 \frac{Q_1}{4\pi\epsilon r} = \frac{1}{2} \cdot \frac{Q_1 Q_2}{4\pi\epsilon r}$$

$$W_e = W_{e1} + W_{e2}$$

$$\boxed{W_e = \frac{Q_1 Q_2}{4\pi\epsilon r}}$$

Poisson's & Laplace equ's

Date

(103)

$$\nabla \cdot \vec{D} = \rho \quad \text{--- Gauss' law}$$

$\vec{D} \downarrow \epsilon \vec{E}$

$$\Rightarrow \epsilon \nabla \cdot \vec{E} = \rho$$

$\vec{E} \downarrow \underbrace{= -\nabla V}_{= -\text{Grad } V}$

$$-\epsilon \underbrace{\nabla \cdot \nabla V}_{= \nabla^2 V} = \rho$$

$$\Rightarrow \boxed{\nabla^2 \cdot V = -\frac{\rho}{\epsilon}} \quad \text{--- poissons equ.}$$

if $\rho = 0$

--- charge free region

$$\boxed{\nabla^2 V = 0} \quad \text{--- laplace equ.}$$

find V

find $\vec{E} = -\nabla V$

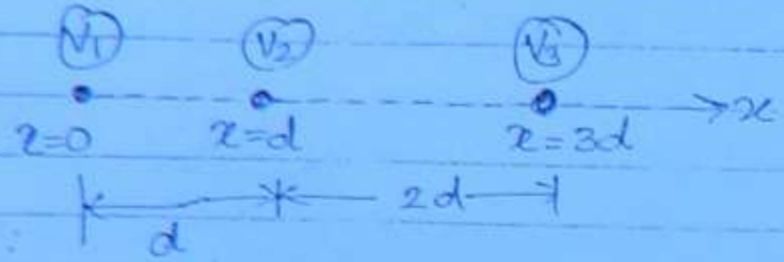
points :

(1) The poissons equn represt. II^{end} order 3-dimen non homogeneous differential eqn.

(2) The Laplace equn rept. 3-dimen. II^{end} order homogeneous differial equ.

(3) using thease equs. the potential V & Elel. field $\vec{E}$ can be found due to a specified volume charge distribution or a charge free Region.

Ex:



104

<u>To find</u> :-

Relation b/w
V_1, V_2, V_3

$\nabla^2 V = 0$

$V = f(x, y, z)$

--- In general

$\boxed{V = f(x)}$ only

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

$\frac{\partial V}{\partial x} = A$; $\boxed{V = Ax + B}$

<u>Boundary conditions</u> :-

1. $x = 0$; $V = V_1$
2. $x = d$; $V = V_2$ just
3. $x = 3d$; $V = V_3$

(1) $V_1 = A0 + B \Rightarrow B = V_1$

(2) $V_2 = Ad + B = Ad + V_1 \Rightarrow A = \frac{V_2 - V_1}{d}$

$V = Ax + B$

Date

$$\boxed{V = \frac{V_2 - V_1}{d} \cdot x + V_1}$$

105

(3)
$$V_3 = \frac{V_2 - V_1}{\cancel{d}} \cdot 3\cancel{d} + V_1$$

$$\boxed{V_3 = 3V_2 - 2V_1}$$ Ans

ex: $V = 3x^4y^4$

To find:
$\rho(1,1,1)$

$$\nabla^2 V = -\rho/\epsilon$$

$$\Rightarrow \rho = -\epsilon \nabla^2 V$$

$$\rho = -\epsilon \left[\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} \right]$$

$$\frac{\partial V}{\partial x} = 12x^3y^4 \quad ; \quad \frac{\partial^2 V}{\partial y^2} = 36x^4y^2$$

$$\frac{\partial^2 V}{\partial x^2} = 36x^2y^4$$

$$\rho = -\epsilon \left[36x^2y^4 + 36x^4y^2 \right]$$

$$\rho(1,1,1) = -\epsilon[72]$$

$$\dots C/m^3$$

ex: $\vec{E} = 2x^2y^2\hat{a}_z \longrightarrow E_z\hat{a}_z$

to find $\rho(1,1,1)$

so $E_x = 0$
$E_y = 0$
$E_z = 2x^2y^2$

$\vec{E}$ Given
$\downarrow$
$\vec{D} = \epsilon\vec{E}$
$\downarrow$
$\rho = \nabla \cdot \vec{D}$

$\vec{D} = \epsilon\, 2x^2y^2\,\hat{a}_z \equiv D_z\,\hat{a}_z$

106

$\rho = \nabla\cdot\vec{D}$

$= \cancel{\dfrac{\partial D_x}{\partial x}} + \cancel{\dfrac{\partial D_y}{\partial y}} + \dfrac{\partial D_z}{\partial z}$

$= \dfrac{\partial D_z}{\partial z} = \dfrac{\partial}{\partial z}\left[\epsilon\cdot 2x^2y^2\right] = 0$

$\nabla\cdot\vec{D} = \rho = 0$

--- Charge free Region

--- $\vec{D}$ is solenoidal

Ex: $V = 4x^2y$; To find $\vec{E}$

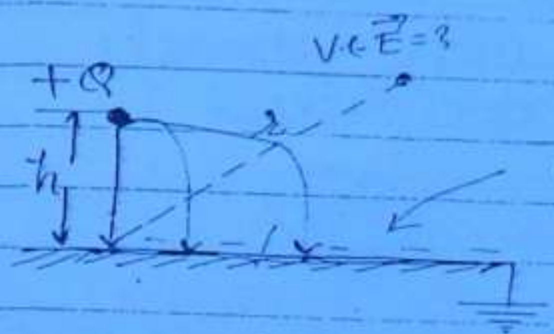
$\vec{E} = -\nabla V = -\text{Grad } V$

$= -\left[\dfrac{\partial V}{\partial x}\hat{a}_x + \dfrac{\partial V}{\partial y}\hat{a}_y + \cancel{\dfrac{\partial V}{\partial z}\hat{a}_z}\right]$

$= -\left[8xy\,\hat{a}_x + 4x^2\,\hat{a}_y\right]$

Method of Image

------ Another method to find V & $\vec{E}$

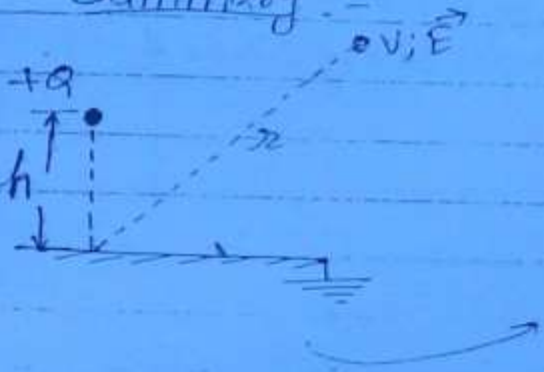


$Q_{ind} = -Q$

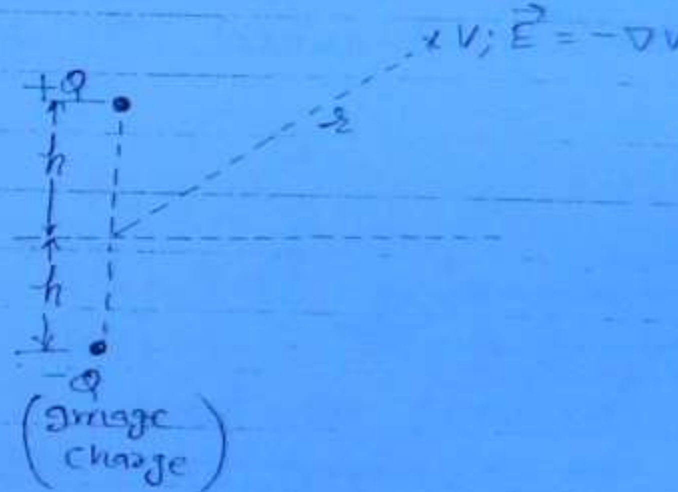
$\boxed{\rho_s\text{ (ind.) is unknown}}$

Infinite cond.
or
grounded cond.

Summary :-



Theory of images



points :-

(1) The method of images is applicable to the e.m. problems where any point charge is placed at some height above an infinite cond. or a grounded cond.

(2) Any perfect conductor behaves as a perfect reflector & therefor acts as electromag. mirror.

The entire theory of optics is an applicable.

(3) . The theory of images is applicable only

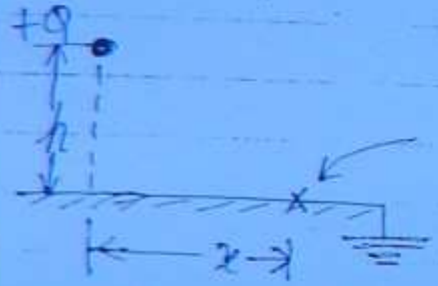
to the e.m. problems.

for mag. field the theory is not applicable. since the mag. charges in the isolated form do not exist in nature.

108

Ex: A point charge $+Q$ is placed at a Height H above a grounded conductor.

Calculate the surface charge density induced on the conducting sheet.

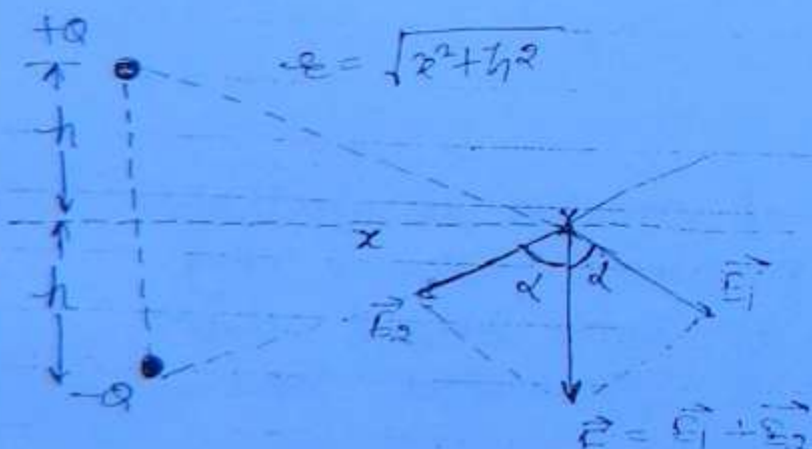


$\vec{E}$

$\vec{D} = \epsilon \vec{E}$

$D_n = |\vec{D}|$

$= |\rho_s|$



$$|\vec{E_1}| = |\vec{E_2}| = E_0 = \frac{Q_0}{4\pi\epsilon r^2}$$

$$E_n = |\vec{E_1}| \cos\alpha + |\vec{E_2}| \cos\alpha$$

$$= 2E_0 \cos\alpha$$

Date

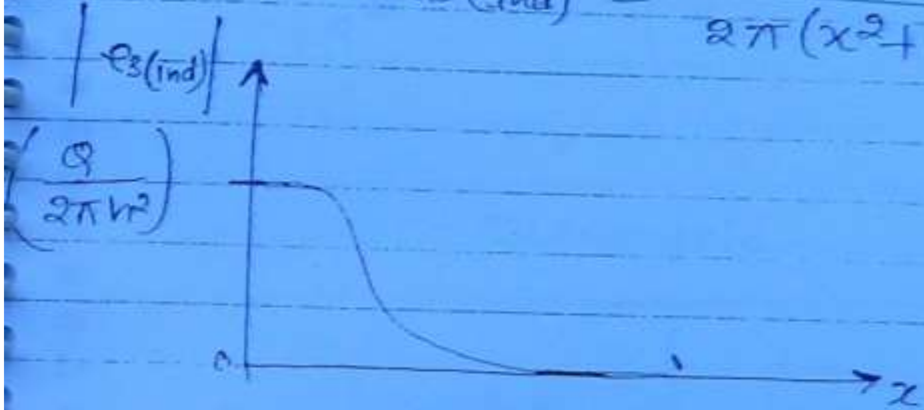
$$= 2\frac{Q}{4\pi\epsilon r^2}\cdot\frac{h}{r}$$

$$\boxed{E_n = \frac{Qh}{2\pi\epsilon(x^2+h^2)^{3/2}}}$$

109

$$D_n = \epsilon E_n \equiv |\rho_s| = \frac{Qh}{2\pi(x^2+h^2)^{3/2}}$$

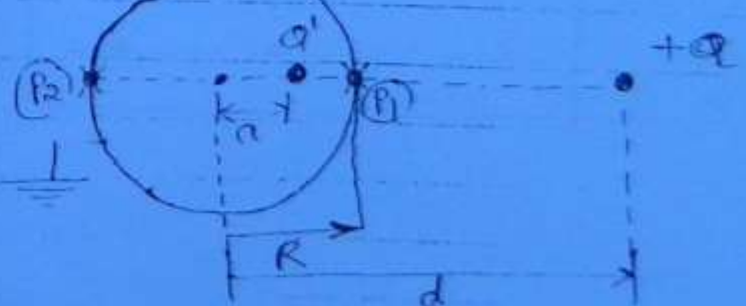
$$\rho_{s(ind)} = \frac{-Qh}{2\pi(x^2+h^2)^{3/2}}$$



Ex: A point charge $+Q$ is placed in front of of a grounded spherical conductor of Radius 'R' as shown.

calculate the Magnitude & the Location of image charge.

Images in spheres



Q^1 = Magnitude } of image
a = location } charge

110

$$V = \frac{Q}{4\pi\epsilon r}$$

$$V_{P1} = \frac{1}{4\pi\epsilon}\left(\frac{Q}{d-R} + \frac{Q^1}{R-a}\right) = 0$$

$$V_{P2} = \frac{1}{4\pi\epsilon}\left(\frac{Q}{d+R} + \frac{Q^1}{R+a}\right) = 0$$

$$\frac{Q}{Q^1} = -\frac{d-R}{R-a}$$

$$\frac{Q}{Q^1} = -\frac{d+R}{R+a}$$

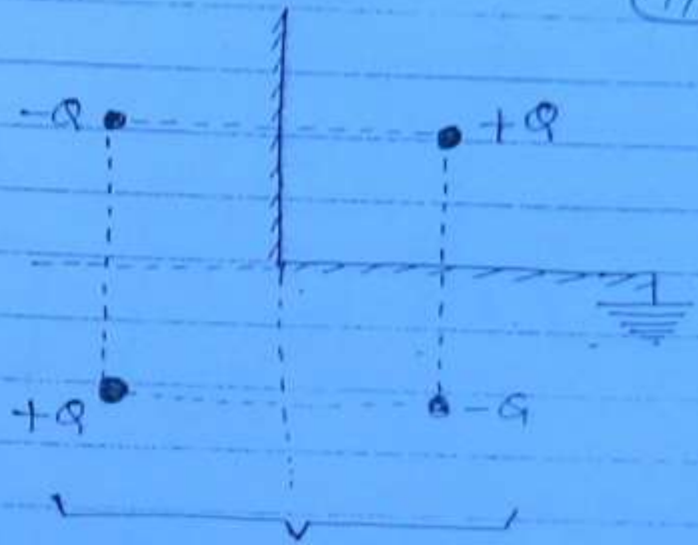
$$\Rightarrow \frac{d-R}{R-a} = \frac{d+R}{R+a}$$

$$\Rightarrow \boxed{a = \frac{R^2}{d}}$$

$$\frac{Q}{Q^1} = -\frac{d-R}{R-a}$$

$$\boxed{Q^1 = -\frac{QR}{d}}$$

ex:



no. of images

$$\boxed{n = \frac{360^\circ}{\theta} - 1}$$

$$n = \frac{360}{90} - 1$$

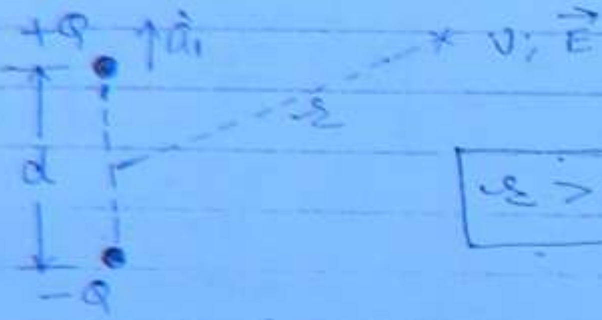
$$= 3$$

--- Quadrupole

type of configuration	$\vec{V}$	$\vec{E}$
Q ● monopole	$1/r$	$1/r^2$
+Q ● ● −Q dipole	$1/r^2$	$1/r^3$
− ● ● + ● + ● − quadrupole	$1/r^3$	$1/r^4$
 octopole	$1/r^4$	$1/r^5$

Main points :-

① elect. dipole



$\boxed{r >> d}$

dipole moment

$$\boxed{\vec{m} = Qd \cdot \hat{a}_1}$$

--- Cm

② $\vec{E}$ due to infinite line charge

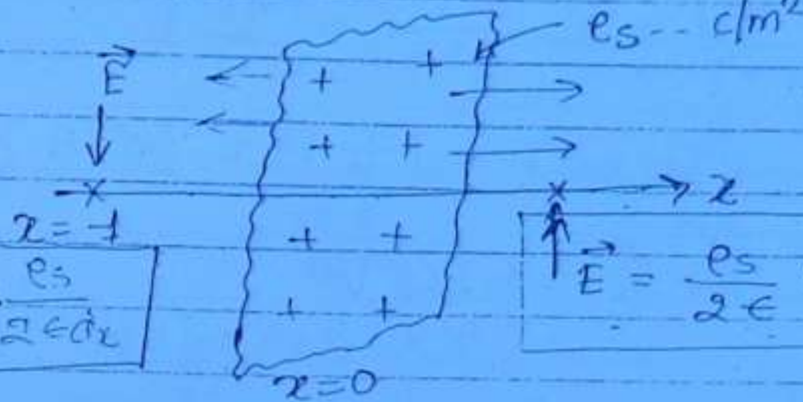
$$\boxed{\vec{E} = \frac{\rho_L}{2\pi\epsilon r} \cdot \hat{a}_r}$$

③ circular loop

+ ρ_L -- C/m

$$\begin{cases} \vec{V} = \text{const.} \\ \vec{E} = 0 \end{cases}$$

(4) Infinite charge sheet :-



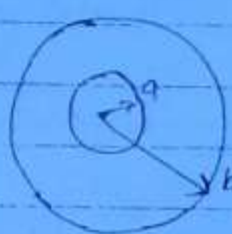
$$\vec{E} = -\frac{\rho_s}{2\epsilon}\hat{a}_x$$

$$\boxed{\vec{E} = \frac{\rho_s}{2\epsilon}\hat{a}_x}$$

$$\rho_s = D_n = \epsilon E_n$$

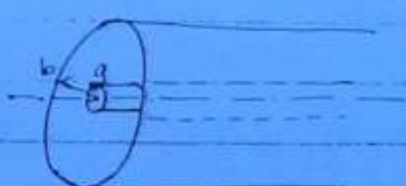
$$E_n = \frac{\rho_s}{2\epsilon}$$

(5) Concentric spherical sheets :-



$$\boxed{C = \frac{4\pi\epsilon ab}{b-a}}$$

(6) cyl. Transmission line :-



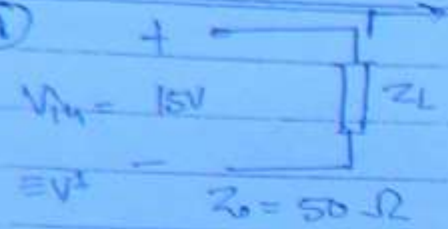
$$\boxed{C = \frac{2\pi\epsilon}{\ln(b/a)}} \text{ F/m}$$

Solution Chp: 3 (T.L.)

Date

(114)

Q: (1)



$\Gamma = 0.268$

$= \rho$

$Z_L = Z_0 \dfrac{1+\Gamma}{1-\Gamma} = \underline{86.6\,\Omega}$

$P_L = \dfrac{V_L^2}{2Z_L}$

$= 2.08\,W$

$\dfrac{V_L}{V^+} = T = 1 + \Gamma = \underline{1.268}$

$\Rightarrow V_L = 1.268\,V^+$

$= 15 \times 1.268 = \underline{19V}$

(2) $Z_0 = \sqrt{\dfrac{L}{C}}$

$v_p = 1/\sqrt{LC}$

$Z_0 v_p = \dfrac{1}{C}$

$v_p = \dfrac{c}{\sqrt{\epsilon_r}}$

$Z_0 \times \dfrac{v}{\sqrt{\epsilon_r}} = \dfrac{1}{C}$

$Z_0 = \dfrac{\sqrt{\epsilon_r}}{v \cdot C}$

(7) $f = 10\,GHz$

$d = 3 \times 10^{-3} m$

$\theta = 90^\circ = \pi/2$

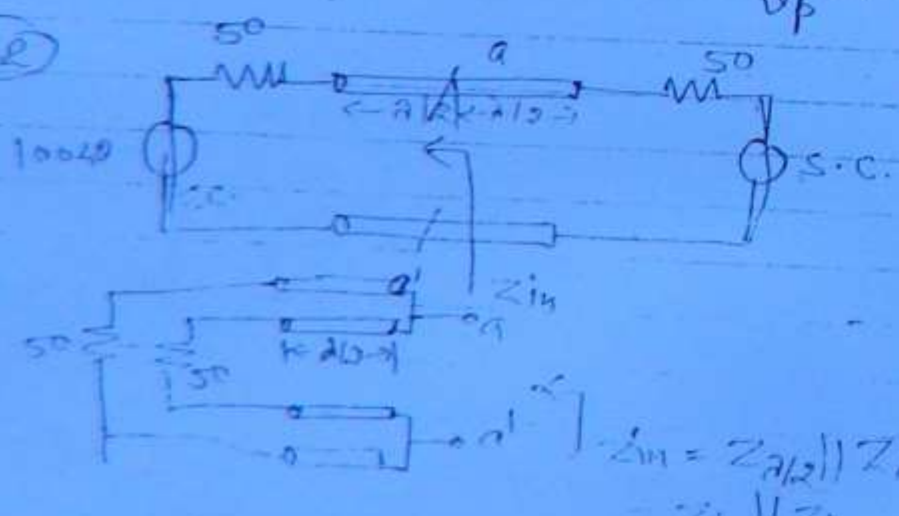
$\longrightarrow$ θ — phase shift — rad

β — phase const — rad/m

$\theta = \beta \cdot d$

$\theta = \dfrac{\omega}{v_p} \cdot d = \dfrac{2\pi \cdot f d}{c/\sqrt{\epsilon_r}}$

(12)



$Z_{in} = Z_{\lambda/2} \,||\, Z_{\lambda/2}$

$Z_L \,||\, Z_L$

$50 \,||\, 50$

$= 25\,\Omega$

(20)

(21) $\lambda/2 = \dfrac{27.5 - 12.5}{15\text{ cm}} = \quad \lambda = 30$ cm

$f = \dfrac{c}{\lambda} = \dfrac{3\times10^{10}}{30} = 1$ GHz

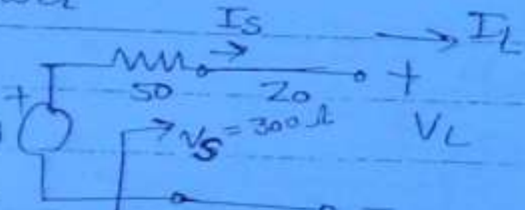
(24) $S = 3$

$\rho = \dfrac{V^-}{V^+} = \dfrac{E_r}{E_i} = \dfrac{S-1}{S+1} = 1/2$

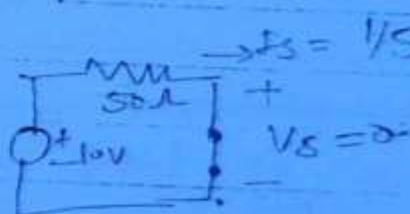
$\underbrace{\rho^2}_{} = 1/4 \Rightarrow 25\%$

reflection coff of power

(26)



$|V_L| = ?$



$Z_{in} = \dfrac{Z_o^2}{Z_L} = \infty$

$Z_{in} = 0$

$V_L = \cos\beta l \cdot \underset{=0}{V_S} - j \underset{300}{Z_o} \underbrace{\sin\beta l}_{\sin\left(\frac{2\pi}{\lambda}\cdot\frac{\lambda}{4}\right) = 1} \cdot \underset{1/5}{I_S}$

$|V_L| = \left| -j\dfrac{300}{5} \right|$

$= 60$ V

(28) $Z_o = \sqrt{Z_{oc}\cdot Z_{sc}}$

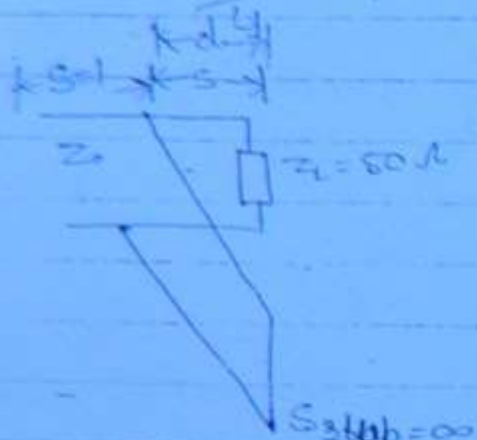
$Z_{sc} = \dfrac{Z_o^2}{Z_{oc}} \leftarrow \underbrace{100 + j150}_{\text{Inductive}}$

(22) $S = \frac{V_{max}}{V_{min}} = \frac{4}{1} = 4$

$Z_{min} = \frac{Z_0}{S}$; $Z_{max} = Z_0 \cdot S$

$= \frac{50}{4} = 12.5\,\Omega$

(16)

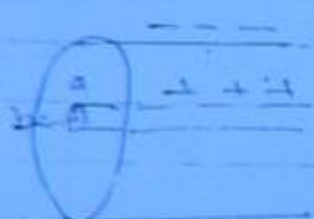


$S = \frac{1+\rho}{1-\rho} = \frac{1+1/3}{1-1/3} = \frac{4}{3} \times \frac{3}{2} = 2$

$\rho = |\Gamma| = 1/3$

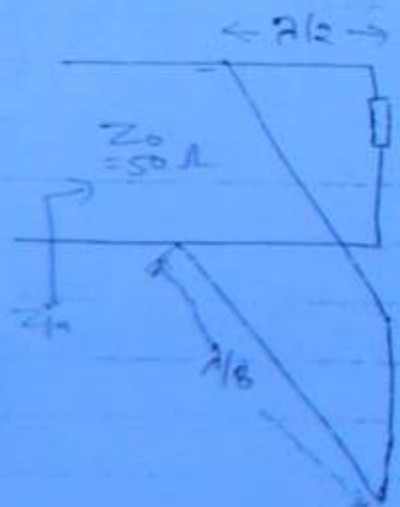
$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{5-10}{5+10} = -1/3$

(15)



$a < r < b$

(25)



$Z_{in} = Z_{\lambda/2} \parallel Z_{\lambda/8}$

$= Z_L \parallel jZ_0 \underbrace{\tan \beta l}_{=1}$

$Z_{in} = Z_L \parallel jZ_0$

$= 100 \parallel 50$

$Y_{in} = Y_L + Y_{\lambda/8}$

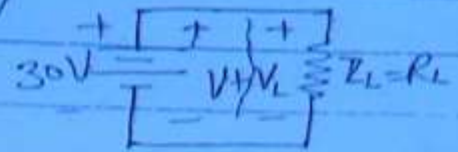
$= \frac{1}{100} + \frac{1}{j50}$

(27) $\vec{H} = 10 \sin(\underbrace{5 \times 10^4}_{\omega} t + \underbrace{0.001}_{\beta} x + \quad)$

$v_p = \frac{-\omega}{\beta}$

Date

(30)



$V_L = V^+ + V^- = 40$

$V^+ = 30$

$V^- = 10$

(1/2)

$I_{ss} \approx \frac{1}{2}\left(I_{max} + I_{min}\right)$

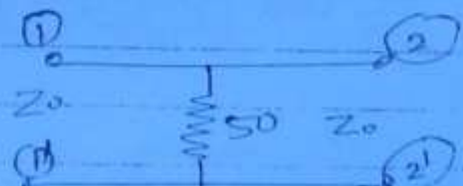
$\frac{V^-}{V^+} = \frac{10}{30} = \frac{1}{3}$; $Z_L = R_L = Z_0 \frac{1+\Gamma}{1-\Gamma} = 100\,\Omega$

$I_{max} = \frac{V_{max}}{R_L} = \frac{V^+ + V^-}{R_L} = \frac{40}{100} = 0.4A$

$I_{min} = \frac{V_{min}}{R_L} = \frac{V^+ - V^-}{R_L} = \frac{20}{100} = 0.2A$

$I_{ss} = \approx \frac{1}{2}(0.4 + 0.2) = 0.3A$

(31)



, $Z_0 = 50\,\Omega$

$[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$

S_{11} = total power sent from port 11' & red. back by port 11' = 0

S_{12} = the total power sent from port 11' & received by port 22' = 1

$[S] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Q: 33. $\frac{\sigma}{\omega\epsilon} << 1$

Good dielect

$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{j\omega\mu}{j\omega\epsilon}} \sqrt{\frac{1}{1 + \frac{\sigma}{j\omega\epsilon}}}$ → ~~small~~

$\frac{1}{\sqrt{1+x}}$

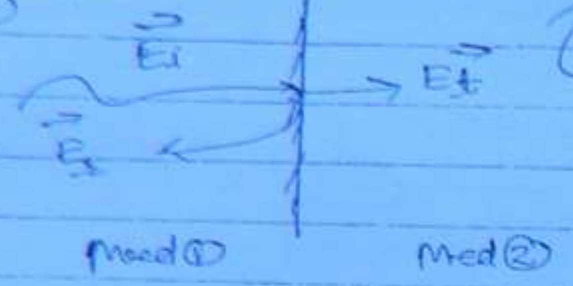
$\eta = \sqrt{\mu/\epsilon}\left(1 + j\frac{\sigma}{2\omega\epsilon}\right)$

Complex   inductive

$\doteq (1+x)^{-1/2}$

$\approx 1 - \frac{1}{2}\frac{\sigma}{j\omega\epsilon} \cdots 1 + j\frac{\sigma}{2\omega\epsilon}$

(38)



(118)

$E_i = E_0 \cos(\omega t - \beta z)\, \hat{a}_y$

$\vec{E_i} = \vec{E_y}\hat{a}_y$
$= E_y \hat{a}_y$

Propagation +z direction; $\vec{E}$ has only y compts.

$\omega = 3\times10^{9}\pi$ — rad/sec.

$\beta = 10\pi$ — rad/m

$v_p = \frac{\omega}{\beta} = 3\times10^{8}$ m/sec.
$= c$

Med (1) is free space

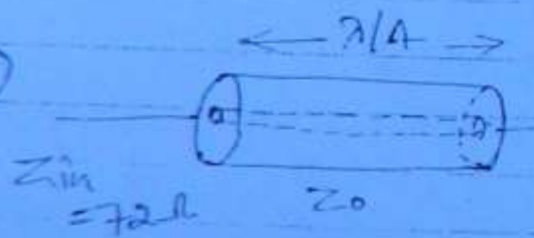
$\vec{E_t} = \ldots \hat{a}_y$
prop. $+z$

Option (c)
$\cancel{\frac{1}{2} E_0 \cdot \cos(\omega t - \beta z)\, \hat{a}_y}$

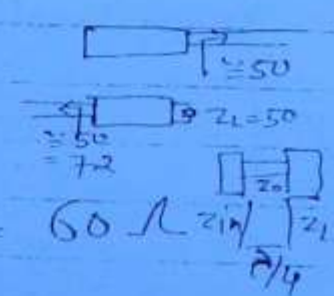
(d)
$\frac{E_0}{-2} \cos(\omega t + 3\beta z)\, \hat{a}_y$

(40) Skin depth $\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$
$\mu \simeq \mu_0$

(42)



$Z_L = 50\,\Omega$



$Z_0 = \sqrt{Z_{in} \cdot Z_L} = \sqrt{72\times50} = 60\,\Omega$

$Z_0 = 60 \ln(b/a) = 60$

$Z_0 = \ln(b/a) \Rightarrow b/a = e^1 \simeq 2.7$

$\frac{2b}{2a} \simeq 2.7$; $2b \simeq (2.7)\underbrace{2a}_{10mm}$

$2b = 27$ mm

Date

44

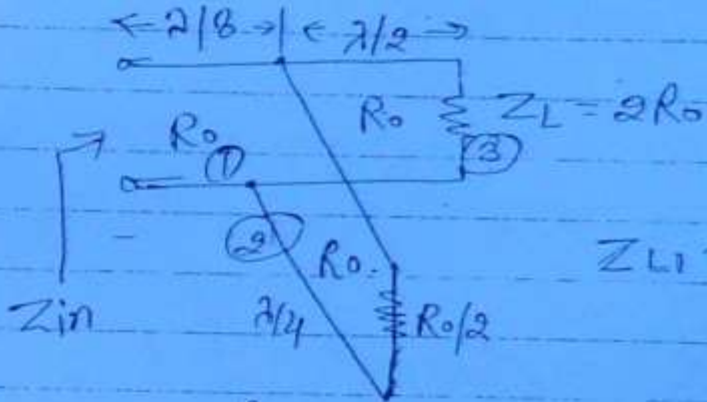
$$\gamma = \underbrace{0.0005}_{\alpha} + j\underbrace{\frac{\pi}{10}}_{\beta}$$

$$\alpha l = \underbrace{(0.0005 \times 50)}_{\times 8.686} \text{ -- nepers}$$

$$\Downarrow$$

-- dB

45



$$Z_{L1} = Z_{\lambda/2} \,||\, Z_{\lambda/4}$$
$$= Z_{L3} \,||\, \frac{Z_0^2}{Z_{L2}}$$
$$= 2R_0 \,||\, \frac{R_0^2}{R_0/2}$$
$$= 2R_0 \,||\, 2R_0 = R_0$$

$$Z_{in}\Big|_{\lambda/8} = Z_0 = R_0$$

perfectly Matched

49 R; G ≅ small

$$\gamma = \alpha + j\beta$$
$$= \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\boxed{\begin{matrix} R << j\omega L \\ G << j\omega C \end{matrix}} = \sqrt{j\omega L}\cdot\sqrt{j\omega C}\left[\left(1 + \frac{R}{j\omega L}\right)\left(1 + \frac{G}{j\omega C}\right)\right]^{1/2}$$

$$j\omega\sqrt{LC}\left[1 + \underbrace{\left(\frac{R}{j\omega L} + \frac{G}{j\omega C}\right)}_{(1+x)^{1/2}} - \frac{RG}{\omega^2 LC}\right]^{1/2}$$

$$= 1 + \frac{1}{2}x$$

$$\gamma = j\omega\sqrt{LC}\left[1 + \frac{1}{2}\left(\frac{R}{j\omega L} + \frac{G}{j\omega C}\right)\right]$$

Real part

$$\alpha = \omega\sqrt{LC}\left[\frac{R}{2\omega L} + \frac{G}{2\omega C}\right]$$ (120)

$$\alpha = \frac{1}{2}\left[R\underbrace{\sqrt{\frac{C}{L}}}_{=1/Z_0} + G\underbrace{\sqrt{\frac{L}{C}}}_{=Z_0}\right]$$

$$\alpha = \frac{1}{2}\left(\frac{R}{Z_0} + GZ_0\right)$$ Ans.

(46) $l = 0.5\,m$

$L =$

$C =$

$f = 25\,MH$

$$\beta l = \frac{\omega}{v_p}\, l = \frac{2\pi f l}{1/\sqrt{LC}} = 2\pi\sqrt{LC}\cdot f\cdot l.$$

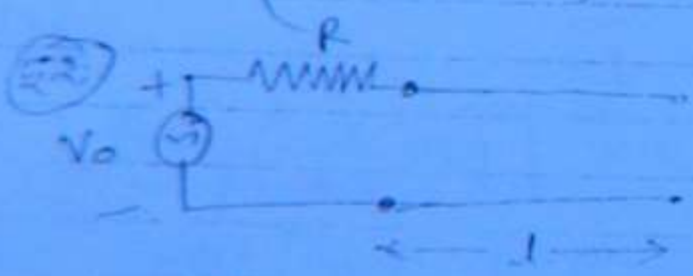
(51) $Z_0 = 50\,\Omega$

$\overline{Z} = 0.5 - j0.3 = \frac{Z}{Z_0}$

$Z = (0.5 - j0.3)\,Z_0$

in smith chart all the imp. are taken in normillased form ($Z = \overline{Z}$)

(22)



$\underline{\underline{V_0}}$

(2) $d = \frac{\lambda}{4} = \frac{1}{4}\frac{v_p}{f} = \frac{1}{4f}\frac{c}{\sqrt{\epsilon_r}}$

$\simeq 0.28$

(3) $\eta = \sqrt{\frac{\mu}{\epsilon}}$ $\qquad v_p = \frac{1}{\sqrt{\mu\epsilon}}$

(6) free space --- perfect diel.

$\sigma = 0$

$\epsilon = \epsilon_0 \underbrace{\epsilon_r}_{1}$ $\qquad \mu = \mu_0 \underbrace{\mu_r}_{=1}$; $\mu_r = 1$; $\mu = \mu_0$

$\epsilon = \epsilon_0$

$\vec{J}_c = \sigma\vec{E} = 0$; $\vec{J}_c = 0$

$\boxed{\rho = 0}$

$v_p = c = 3\times 10^8$ m/sec.

$\eta = \eta_0 = \sqrt{\mu_0/\epsilon_0} = 120\pi\ \Omega$

(11) $\vec{H} = 0.1 \sin(10^8\pi t + \underbrace{\beta y}) \hat{a}_z$ $\longrightarrow$ prop. $-\hat{a}_y$

$\vec{P} = \frac{1}{2}\eta_0 H_m^2$

$\frac{1}{2}\times 120\pi \times 0.1$

(15) $\sigma = 5$ mho/m $\qquad E_z = E_0 e^{-\alpha z}$

$\epsilon_r = 80$ $\qquad e^{-\alpha z} = \frac{E_z}{E_0} = 0.1$

$f = 25$ KHz

--- low freq. $\qquad e^{+\alpha z} = 10$

$\boxed{z = \frac{1}{\alpha}\ln(10)}$

$\frac{\sigma}{\omega\epsilon} >>$ Good Conductor

$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f \mu\sigma}$ $\quad \mu \simeq \mu_0$

(19) E_x

--- sinusoidal time variation

---- prop $\Rightarrow +z$

lossless Meadium

$\sigma = 0$

$\Rightarrow \alpha = 0$

$E_x = E_0 e^{-\gamma z}$

$E_x = E_0 e^{-(\alpha + j\beta) z}$ (T22)

$E_x = E_0 e^{-j\beta z}$

$= E_0 e^{-jkz}$

(20) $\vec{E} = 50 \sin(\underset{\uparrow \omega}{10^7 t} + \underbrace{Kz}) \hat{a}_y = E_y \hat{a}_y$

Prop $= -z$

$\vec{P} = -P_z \hat{a}_z$

$K = \beta = \dfrac{\omega}{v_p} = \dfrac{10^7}{3\times 10^8} = \dfrac{1}{30}$

$\lambda = \dfrac{2\pi}{\beta} = 2\pi \times 30 = 60\pi \simeq 188.5\,m$

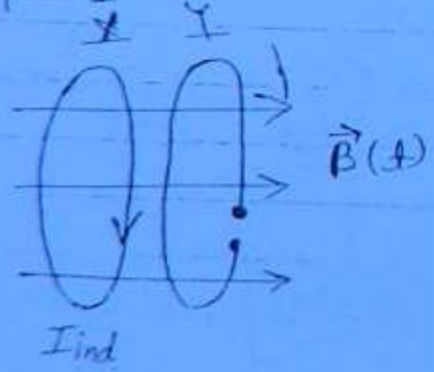
(21) $\mu = \epsilon$; $\eta_1 = \eta_0 = 377\,\Omega$

$\eta_2 = \sqrt{\dfrac{\mu}{\epsilon}} = 1\,\Omega$

$\rho = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \dfrac{1 - 377}{1 + 377} = -\dfrac{376}{378} \simeq -1$

$|\rho| \simeq 1$

(22)

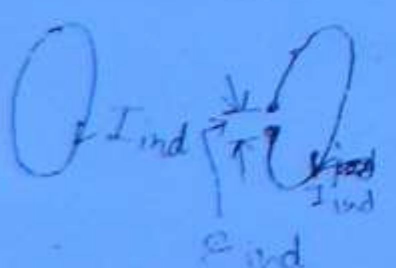


$\vec{B}(t) \rightarrow \psi_m(t) \rightarrow \dfrac{\partial \psi_m}{\partial t} \rightarrow e_{ind}$

$\hookrightarrow I_{ind}$

given by Lenz Law

faraday's Law of emi



Comments :- using faraday's laws of e.m induction, due to rate of change of Mag flux induced current is produced in the circular loop which will ride to

Induced emf. when the loop is open-circuited.
Therefor Induced emf equivalent effect of I_{ind} is always present on the loop when ever mag. field is time variant.

(24) $\vec{A} = \frac{\mu}{4\pi}\int_c \frac{\vec{I}}{r} dl$ — 123

--- due to line current.

as $r \to \infty$; $\vec{A} = 0$

(26) $\left.\begin{matrix} v_p \\ \eta \end{matrix}\right\}$ $\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}} = 377\sqrt{\frac{\mu_r}{\epsilon_r}}$

(27) perfect cond. (2) | (1)

$E_{t1} = E_{t2} = 0$

$H_{t1} - H_{t2} = \cancel{J_s} \to \infty$

$H_{t1} = H_{t2} = 0$

(29) $\frac{I_c}{I_d} = \frac{J_c}{J_d} = \frac{\sigma}{\omega\epsilon} = 1$

$\sigma = \omega\epsilon = 2\pi f \epsilon$

$f = \frac{\sigma}{2\pi \epsilon_0 \epsilon_r}$

Wave propagation

Date

① Surface wave propagation :-

--- used at meadium freq.

--- AM broad cast

(124)

f : 535 KHz – 1605 KHz

② space wave prop.

Tropospheric prop } --- used at VHF; microwave range

LOS(line-of-sight)prop

$\underbrace{\text{microwave range}}_{3GHz - 30GHz}$

--- preferred for

–TV broadcast

– microwave link.

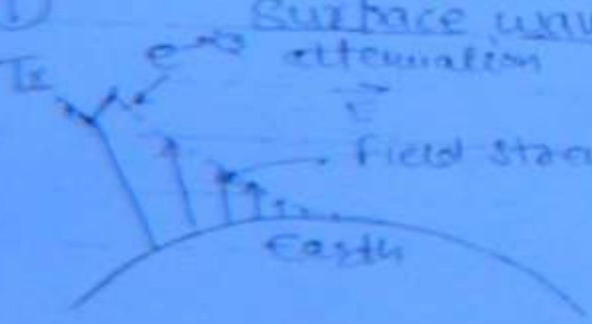
③ sky-wave prop } --- used at
Ionospheric

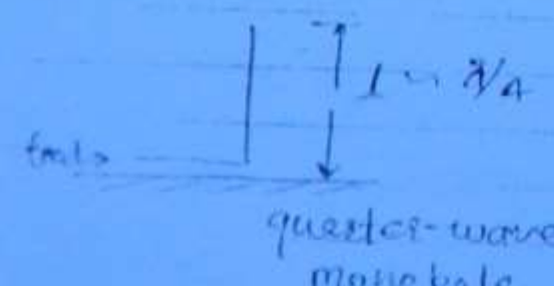
HF rang (short-wave range)

preffered for FM broad cast

(f : 88MHz – 108MHz)

① Surface wave propagation





features :-

① The e.m waves travelles along the surface of the earth.

Date

(2) The transmitting entina is always vertically installed & therefor is always vertically polarized.

125

(3) The elet field associated with the wave is perpendicular to the surface of earth.

(4) The earth behaves as good conductor & therefor as the wave travell the elect. field strength decrease exponentially.

(5) The length of the entina depends upon the freq. or wavelength of operation quarterwave monopole are always preffered for the transmisson of such signal.

(6) as the frequency of operation increase the Height of entina decreases.

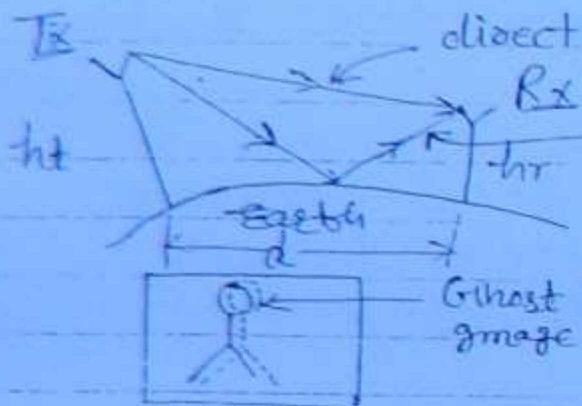
(7) preffered for AM broadcast on the freq. rang 535 to 1605 KHz.
The range of transmisson can be increase only by increasing power of Transmitter.

(8) such propagation take place when the transmitting & receiving entina very closed to each other & close to surface of earth.

(9) Limited range of transmisson

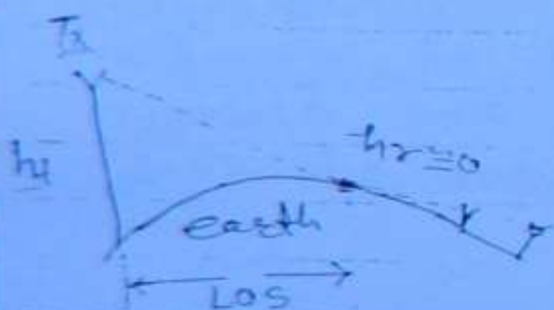
126

(2) Space wave prop.
--- Troposphseric prop.
--- LOS prop.



$$\boxed{d \simeq 3550(\sqrt{h_t} + \sqrt{h_r})}$$

---- m



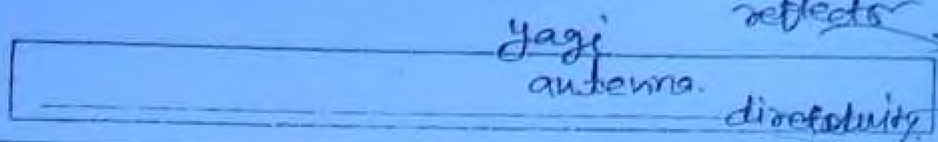
$$\boxed{d \simeq 3550\sqrt{h_t}}$$

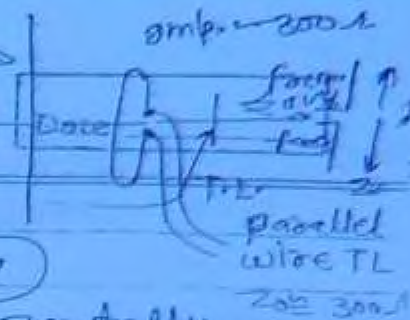
Features :-

(1) The space wave consitutes :
(a) direct wave
(b) ground reflected wave.

(2) preffered for freq. greater than 30MHz for TV broadcast in the VHF range.

) The e.m. wave travel from transmitter to reciever in the earth tropospheric at a height of 10 to 15 km above





The surface of the earth.

T22

(3) The entina is always horizontally polarized so that elect field vector is prallel to the surface of the earth.

(4) foldy pole is preffered for such propg. so that it's impedance is mathed with that of T.L.

(5) The total range of Transmisson depends upon:

1. power of The transmittor.
2. Height of Transmitting & Receiving Antina.
3. The range of Transmisson can be increas by increasing the Height of the receiving entina.

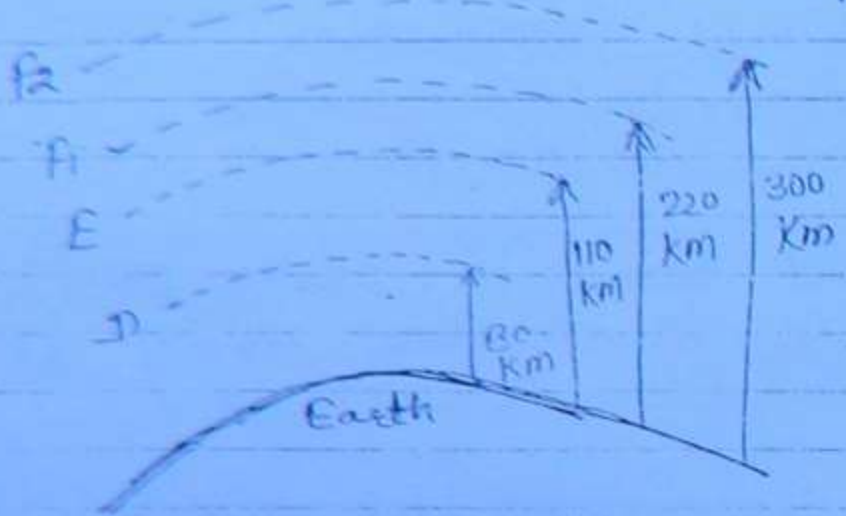
(6) factors which control of the magnitude of space wave:-

(a) Conductivity of earth
(b) permittivity & permibility of earth
(c) freq. of the wave.
(d) Heights of Transmitting & receiving entina
(e) curvature of the earth
(f) Distance b/w T/m & Receiving entina.
(g) Variation of reflecting index of earth with Height.

(3) Sky wave prop

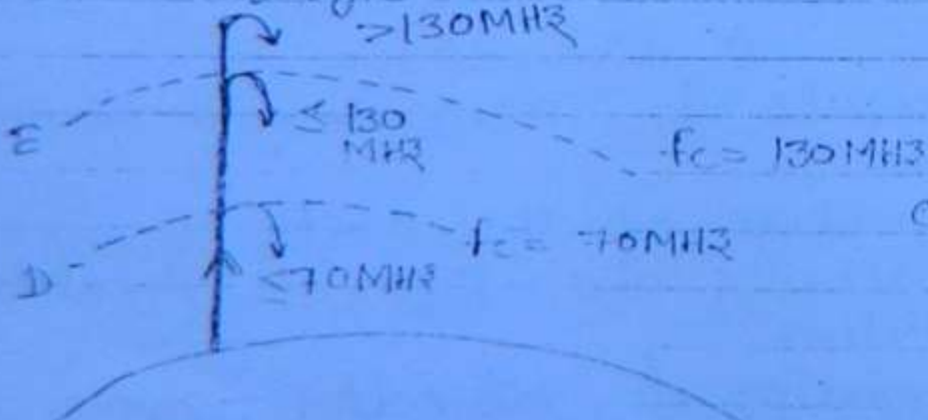
--- Ionospheric prop.

128



θ = TOA

- take of angle



critical freq. of layer

h ht of layer.

S skip distance.

(1) used in the HFR & preffered for FM bradcast in the freq. range 88 MHz to 200 MHz.

(2) long range Transmission is possible.

(3) The range of Transmisson depends:
(a) Take of angle.
(b) freq. of the signal.

(4) The D layer has minimum electron density whereas f2 layer has Max. electron density.

(5) The critical freq. of a layer depends upon the electron density of the layer.

$$\boxed{f_c = \sqrt{81\,N}}$$

critical freq. MHz — electron density ---/m³

Therefor the critical freq. of the D layer is minimum where as this freq. has a Max. value for f2 layer.

(6) During night time d-layer is missing, f1 & f2 layer merge together to form a single layer.

(7) The E-layer is the most stable layer & FM broadcast uses this layer for the T/M of e/c signal.

fc) critical freq angle :- This is the maximum freq. of a layer so that the wave is reflected by that layer at vertical incidence. (130)

The e.m. wave of freq. less than or equal to critical freq. will be reflected from the layer irrespective of the angle of incidence.

as the height of layer increases. its critical freq. increases

S) skip distance :- This is the minimum distance from the Tx at the sky wave of given freq. it return to earth by the ionospheric.

The skip distance depends upon:

(a) freq. of the wave.
(b) fc (critical freq)
(c) Height of the layer.
(d) charge carried concentration + N.

MUF) --- Maximum usable frequency.

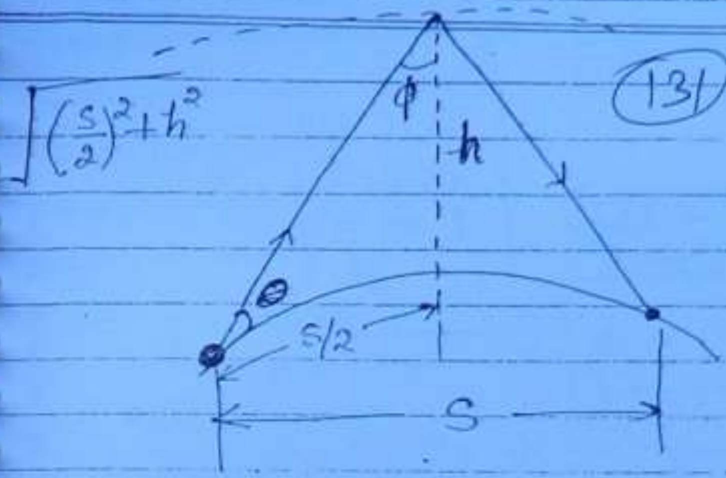
$$\boxed{MUF = f_c \cdot \sec\theta}$$

$MUF > f_c$

$\left\{ \begin{array}{l} Tx = \text{Transmitter} \\ Rx = \text{Receiver} \end{array} \right.$

Date

131



θ = TOA

ϕ = angle of incidence at the layer

$$\boxed{\theta+\phi = 90^\circ}$$

$$\phi = 90^\circ - \theta$$

$$MUF = f_c \cdot \sec\phi$$

$$= f_c \cdot \frac{\sqrt{\left(\frac{s}{2}\right)^2+h^2}}{h}$$

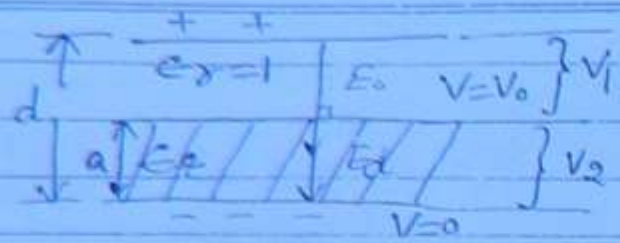
$$\boxed{MUF = f_c \cdot \sqrt{\left(\frac{s}{2h}\right)^2+1}}$$

(1) for fixed location of the Tx & Rx, MUF is the freq. which makes the distance to the reciving point equal to the skip distance.

(2) MUF is the freq. that gives strongest sky wave signal at the recived point.

(3) The skip distance increases with the freq. of operation.

ex:



To find E_0 ; E_d

132

$E = \frac{V}{d}$

$V_1 + V_2 = 0$

$E_0(d-a) + E_d \cdot a = V_0$ —— (1)

$D_{n1} - D_{n2} = \rho_s$

$D_1 = D_2$

$\epsilon_0 E_0 = \epsilon_0 \epsilon_r E_d$

$\boxed{E_0 = \epsilon_r E_d}$ —— (2)

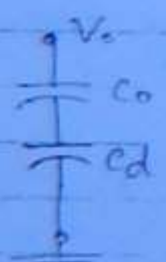
$\epsilon_r E_d (d-a) + E_d\, a = V_0$

$E_d [\epsilon_r d - \epsilon_r a + a] = V_0$

$$E_d = \frac{V_0}{\epsilon_r(d-a) + a}$$

$$E_0 = \epsilon_r \cdot E_d = \frac{\epsilon_r V_0}{\epsilon_r(d-a) + a} = \frac{\epsilon_r V_0}{\epsilon_r(d-a) + a}$$

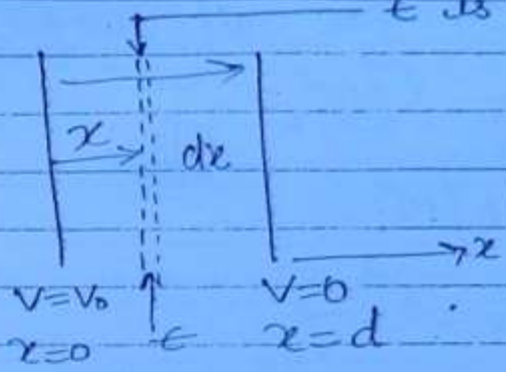
C =



$C = \frac{C_0 C_d}{C_0 + C_d}$; $C_0 = \frac{\epsilon_0 A}{d-a}$

$C_d = \frac{\epsilon_0 \epsilon_r A}{a}$

ex



ϵ is varying linearly w.r.t x

at $x=0$; $\epsilon = \epsilon_1$ -- min.

$x=d$; $\epsilon = \epsilon_2$ -- Ma

133

To find Capacitance

$$\epsilon = \epsilon_1 + \frac{\epsilon_2 - \epsilon_1}{d} x$$

$\rho_s = D_n$

$V = -\int_{x=d}^{0} \vec{E} \cdot d\vec{l}$

$\vec{E} = -\nabla V$

$$= -\int_{x=d}^{0} E_n \cdot dx \Rightarrow -\int_{x=d}^{0} \frac{D_n}{\epsilon} \cdot dx$$

$$= -\int_{x=d}^{0} \frac{\rho_s}{\epsilon_1 + \frac{\epsilon_2 - \epsilon_1}{d} \cdot x} dx$$

$\epsilon_1 + \frac{\epsilon_2 - \epsilon_1}{d} x = t$

$\frac{\epsilon_2 - \epsilon_1}{d} dx = dt$

$$= -\rho_s \int_{x=d}^{0} \frac{1}{t} \cdot \frac{d}{\epsilon_2 - \epsilon_1} dt$$

$$= \frac{-\rho_s \cdot d}{\epsilon_2 - \epsilon_1} \ln t \Big|_{x=d}^{0}$$

$$= \frac{-\rho_s d}{\epsilon_2 - \epsilon_1} \ln \left[\epsilon_1 + \frac{\epsilon_2 - \epsilon_1}{d} x \right]_{x=d}^{0}$$

$$= \frac{-\rho_s \cdot d}{\epsilon_2 - \epsilon_1} \ln \frac{\epsilon_1}{\epsilon_2}$$

$$= + \frac{\rho_s \cdot d}{\epsilon_2 - \epsilon_1} \ln \left(\frac{\epsilon_2}{\epsilon_1} \right)$$

$C = \frac{Q}{V} = \frac{\rho_s}{V}$ -- Cap. per unit surface

F/m^2

$$\boxed{C = \frac{\epsilon_2 - \epsilon_1}{d \ln\left(\frac{\epsilon_2}{\epsilon_1}\right)}} \quad F/m^2$$

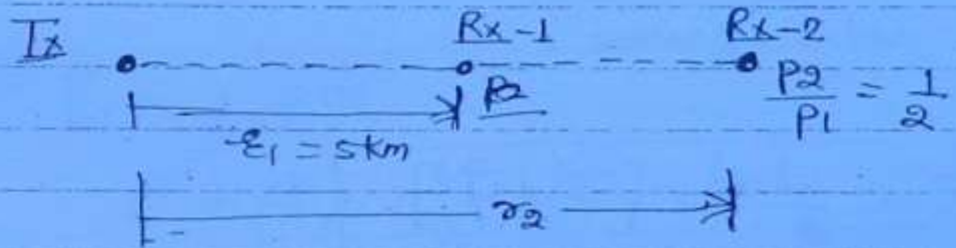
30. $\tan \delta = \dfrac{\sigma}{\omega \epsilon}$

$\omega = 2\pi f$, $\epsilon = \epsilon_0 \epsilon_r$

31. $\rho_s = D_n = \epsilon E_n$

$\epsilon = 80\epsilon_0$, $E_n = 2\,V/m$

32.



$d = r_2 - r_1$
$= (r_2 - 5)\,km$

$|\vec{P}| = \dfrac{power}{4\pi r^2}$

$|\vec{P}| \propto \dfrac{1}{r^2} = P$

$$\left(\frac{P_2}{P_1}\right) = \left(\frac{r_1}{r_2}\right)^2 = \frac{1}{2}$$

$\Rightarrow r_2 = \sqrt{2}\, r_1 \Rightarrow 5\sqrt{2}\ km$

$\Rightarrow r_2 - r_1 = d = (5\sqrt{2} - 5) = 5(\sqrt{2} - 1)$
$= 2.070\ km$

$P_{dB} = 10 \log_{10} P$

$$\left(\frac{P_2}{P_1}\right) = \underbrace{-3\,dB \equiv 10 \log_{10}\left(\frac{P_2}{P_1}\right)}$$

$$\Rightarrow \frac{P_2}{P_1} = \frac{1}{2}$$

(37) $\epsilon = \epsilon_0$

A

d

$v = 0.5\,V$

$f = 3.6\,GHz$

$I_d = J_d \cdot \vec{A} = \frac{\partial D}{\partial t} \cdot A = j\omega \epsilon E A = j\omega \epsilon_0 \frac{v}{d}$

$\uparrow$

A/m^2

$|I_d| = 2\pi f \epsilon_0 \frac{v}{d} A$

(38)

Tensor:

$$\boxed{\vec{P} = \frac{1}{2} Re\left[\vec{E} \times \vec{H}^*\right]}$$

$$\vec{P} = \frac{1}{2} Re\left[(\hat{a}_z + j\hat{a}_y)\cancel{e^{jkz - j\omega t}} \times \underbrace{\left(\frac{K}{\omega\mu}\right)(\hat{a}_y - j\hat{a}_x)\cancel{e^{-jkz + j\omega t}}}_{\vec{H}^*}\right]$$

$K' \, Re\left[\hat{a}_z - \hat{a}_z\right]$

$= \vec{0}$ -- null vector.

$\hat{a}_x \to \hat{a}_y \to \hat{a}_z$

① $\vec{P} = \vec{E} \times \vec{H}$ -- Poynting's vector

② $\vec{P} = \frac{1}{2} \vec{E} \times \vec{H}$ -- average poynting vector.

③ $\vec{P} = \frac{1}{2} Re\left[\vec{E} \times \vec{H}^*\right]$ ---- average poynting vector when $\vec{E} \times \vec{H}$ are phasors.

ex.

Date

(1) $\vec{E} = 2\hat{a}_x$

$\vec{H} = 4\hat{a}_y$

137

$\vec{P} = \vec{E} \times \vec{H} = 8\hat{a}_z$

(2) $\vec{E} = 4\cos(\omega t - \beta z)\hat{a}_x$

$\vec{H} = 2\cos(\omega t - \beta z)\hat{a}_y)$

$\vec{P} = (\vec{E} \times \vec{H}) = 8\underbrace{\cos^2(\,..\,)}_{\text{average value}}\hat{a}_z$

$\cos^2\theta = \frac{1}{2}(1 - \underbrace{\cos 2\theta}_{0}$

$\vec{P}_{av} = \frac{1}{2} 8\hat{a}_z$

(3) $\vec{E} = \underbrace{(\hat{a}_x + j a_y)}_{\text{phasor}} e^{jkz - j\omega t}$

$\vec{H} = \text{-- -- -- --}$

Q : 39 Med (1) -- free space

$\eta_0 ; \epsilon_0$

Med (2) ---- perfect diele. $(\sigma = 0 ; \mu_r = 1)$

$\eta_2 ; \epsilon_2 : \eta ; \epsilon$

$$\boxed{\epsilon > \epsilon_0}$$

$$S = \frac{V_{max}}{V_{min}} = \frac{E_{max}}{E_{min}} = 5$$

$$\rho = \frac{S-1}{S+1} = \frac{5-1}{5+1} = \frac{2}{3}$$

$$\rho = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\eta - \eta_0}{\eta + \eta_0} = \underbrace{\frac{\sqrt{\epsilon_0} - \sqrt{\epsilon}}{\sqrt{\epsilon_0} + \sqrt{\epsilon}}}$$

$$\rho = \frac{\eta_0 - \eta}{\eta_0 + \eta} \leftarrow \frac{\sqrt{\epsilon} - \sqrt{\epsilon_0}}{\sqrt{\epsilon} + \sqrt{\epsilon_0}} \leftarrow$$

$\eta_0 = 120\pi$

$\eta = 24\pi$ Ans.

Q: 41. $P_i = P_r + P_t$ (138)

$$1 = \frac{P_r}{P_i} + \frac{P_t}{P_i}$$

$$\left(\frac{P_t}{P_i}\right) = \left[1 - \underbrace{\left(\frac{P_r}{P_i}\right)}\right]$$

← reflection cofficient of powr.

$$\frac{P_t}{P_i} = 1 - \left(\frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}}\right)^2 \quad ; \quad \begin{aligned} \epsilon_1 &= \epsilon_0 \epsilon_{r1} = \epsilon_0 \\ \epsilon_2 &= \epsilon_0 \epsilon_{r2} = 4\epsilon_0 \end{aligned}$$

$$= \frac{8}{9}$$

Q: 42 Imp.

$$\vec{E} \quad E_x = E_0 \underbrace{e^{-\gamma z}}_{\text{along } +z \text{ direction}}$$

$$= E_0 e^{-(\alpha + j\beta) z}$$

$$= E_0 e^{-j\beta z} \quad \text{---- lossless Med.}$$

$$E_0 e^{-j\beta z} \cdot e^{j\omega t} \quad \text{--- for sinusoidally varying field}$$

$$\boxed{E_x = E_0 e^{j(\omega t - \beta z)}}$$

+ z direction prop.

→ Wave propagates along some arbitrary direction

$$E_x = E_0 e^{j(\omega t - \vec{\beta} \cdot \vec{r})}$$

To be inued.

Date

$$\vec{\beta}\cdot\vec{r} = \left(\beta_x \hat{a}_x + \beta_y \hat{a}_y + \beta_z \hat{a}_z\right)\cdot\left(x\cdot\hat{a}_x + y\cdot\hat{a}_y + z\hat{a}_z\right)$$

$$\vec{\beta}\cdot\vec{r} = \beta_x\cdot x + \beta_y\cdot y + \beta_z\cdot z$$

139

$$= \beta\cdot\cos\theta_1 + \beta\cos\theta_2 + \beta\cos\theta_3$$

$$\frac{2\pi}{\lambda}\underbrace{\cos 30^\circ}_{\sqrt{3}/2} \qquad \theta_2 = 90^\circ \qquad \beta\cos 60^\circ$$

$$= 0^\circ \qquad \frac{2\pi}{\lambda}\cdot\frac{1}{2}$$

$$= \frac{\sqrt{3}\cdot\pi}{\lambda} \qquad = \pi/\lambda$$

$$E_z = E_0\, e^{j\left(\omega t - \frac{\sqrt{3}\pi}{\lambda}x - 0 - \frac{\pi}{\lambda}\cdot z\right)}$$

44 Incident -- RCP

reflected -- LP

$\Downarrow$

$$\theta = \theta_B$$

$$\underbrace{\tan\theta_B}_{\sqrt{3}} = \sqrt{\frac{\epsilon_2 \rightarrow \epsilon_{r2}\cdot\epsilon_0}{\epsilon_1 \rightarrow \epsilon_0\epsilon_{r1} = \epsilon_0}}$$

$$\epsilon_{r2} = 3$$

46 $\epsilon_{r1} = 1$ $\qquad \vec{E_1} = 1.\hat{a}_x$

$$\rho_s = D_{n1} - D_{n2}$$

$\epsilon_{r2} = 2$ $\qquad \vec{E_2} = 2\hat{a}_x$

$$= \epsilon_0\epsilon_{r1}E_{n1} - \epsilon_0\epsilon_{r2}E_{n2}$$

$$= \epsilon_0[1\times1 - 2\times2]$$

$$= -3\epsilon_0$$

(49) ε_{ind} -- both coil

140

perfect cond.

$\Downarrow$

losses $\simeq 0$

$\Downarrow$

Heat dissipation $= 0$

$\vec{H} = H_x \hat{a}_x$

(50) $\vec{H} = 0.1 \cos(4 \times 10^7 t - \underbrace{\beta z}) \hat{a}_z$

prop $\Rightarrow +z$

$\vec{P} = \vec{E} \times \vec{H}$ $\qquad \vec{P} = P_z \hat{a}_z$

$\hat{a}_z = (-\hat{a}_y) \times (\hat{a}_x)$

$\hat{a}_x$ & a_z

$\hat{a}_y$ &

a_z

$\vec{E} = -E_y \hat{a}_y$ Notice

$= \eta_0 H_m = 377 \times 0.1$

(51) $\vec{E} = 50 \cos(--) \hat{a}_x$

$\vec{H} = \frac{5}{12\pi} \cos(--) \hat{a}_y$

direct of prop. $\Rightarrow +z$

Power $= P_z \times$ area

$\frac{1}{2} E_m H_m (\pi r^2)$

$\uparrow$ 50 $\quad \uparrow$ $5/12\pi$ $\quad \uparrow$ $\sqrt{24}$ m

Date

(53) $\vec{H} = (\qquad\qquad) \hat{a}_z$

$\vec{E} \neq \hat{a}_z$

-- wave is not polarized in z direction

(54) $\left(\frac{\vec{F}}{l}\right) = \vec{I} \times \vec{B}$

(63) E_{ind} — Cond. is Moving $(\vec{v})$; $\vec{B} \neq f(t)$

— $\vec{v} = 0$; $\vec{B} = f(t)$

→ $\vec{v}$; $\vec{B} = f(t)$

Case 1:- moving cond. ; $\vec{v}$

$\vec{B} \neq f(t)$

$$E_{ind} = \oint_c (\vec{v} \times \vec{B}) \cdot dl$$ ---- Motional emf.

Case 2: $\vec{v} = 0$

$\vec{B} = f(t)$

$$E_{ind} = -\iint_s \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$ --- due to time varying field

Case 3: Moving cond. --- emf due to transformer action

$\vec{B} = F(t)$ ∴ Ans

$E_{ind} = E_{ind}(\text{case 1}) + E_{ind}(\text{case 2})$

(67) σ ; ρ_s $\qquad |D_n| = \rho_s$

$p = w_e = \frac{1}{2} \epsilon E_m^2$ $\qquad$ 142

$= \frac{\epsilon}{2} \left(\frac{D_n}{\epsilon}\right)^2$

$= \frac{1}{2} \cancel{\epsilon} \frac{\rho_s^2}{\epsilon^2} = \frac{1}{2\epsilon} \rho_s^2$

$= \frac{1}{2} \epsilon \sigma^2$

(68) $PE = KE$

potential energy = kinetic energy

$eV = \frac{1}{2} m v^2$

$v = \sqrt{\frac{2eV}{m}}$ ---- Known

$F = \frac{m v^{\cancel{2}}}{r} = e \cancel{v} B$

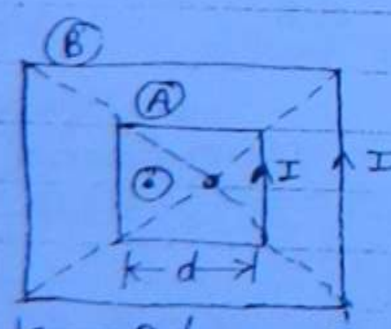
$\boxed{r = \frac{mv}{eB}}$ result.

250 gauss → to convert

250×10^{-4}

--- = --- wb/m^2

(69)



$\frac{H_A}{H_B} = ? = \frac{r_B}{r_A}$ $\qquad H \propto \frac{1}{r}$

$$\frac{\epsilon_B}{\epsilon_A} = \frac{d}{d/2} = 2$$

143

→ sam e nature in $\vec{H}$

(74) $\vec{E} = \underbrace{\hat{a}_y \cdot A}\cos\omega\left(t - \frac{z}{c}\right)$

$\vec{E} = E_y\,\hat{a}_y$ prop. ⇒ $+z$

$\vec{H} = ?$ $\vec{P} = P_z\,\hat{a}_z$

$\vec{P} = \vec{E} \times \vec{H}$

$\hat{a}_z = \hat{a}_y \times \underbrace{(-a_x)}$

$\vec{H} = -H_x\,\hat{a}_x$

$$\frac{E}{\eta} = E_y\sqrt{\frac{\epsilon_0}{\mu_0}}$$

options

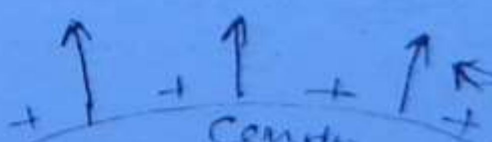
(c) $\hat{a}_x\left[-j\sqrt{\frac{\epsilon_0}{\mu_0}}\,A\cos(\ldots)\right]$ $= \sin(\ldots)$

(d) $\hat{a}_x\left[-j\sqrt{\frac{\epsilon_0}{\mu_0}}\cdot A\,\underbrace{\sin(\ldots)}\right]$ ✓ Ans $= \cos(\ldots)$

(75) $\vec{D} = 2(a_x - \sqrt{3}\,\hat{a}_z)$

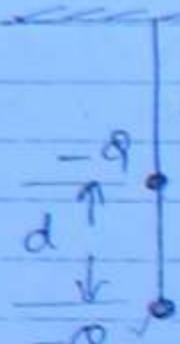
$\rho_s = |D_n| = 2\sqrt{1+3} = 4\ C/m^2$

$|D_n| = \rho_s = +4\ C/m^2$



Date

Q: 2



$$\uparrow f_e = \frac{Q^2}{4\pi\epsilon_0 d^2}$$

144

$-Q$

d

$$\downarrow f_g = mg$$

$-Q$

$$mg = \frac{Q^2}{4\pi\epsilon_0 d^2} \quad \text{find } d = 857 \text{ cm}$$

Q: 3

I-method

$$V = -\frac{6r^5}{\epsilon_0}$$

$\downarrow$

find $\vec{E} = -\nabla V$

$\downarrow$

find $\vec{D} = \epsilon\vec{E}$

$\downarrow$

find $\rho = \nabla\cdot\vec{D}$

$\downarrow$

find $Q = \iiint_V \rho\, dv$

II-method

$\nabla^2 V = -\rho/\epsilon$

find $\rho = -\epsilon_0 \nabla^2 V$

find $Q = \iiint_V \rho\, dv$

$$\rho = -\epsilon_0 \nabla^2 V \; ; \quad V = f(r)$$

$$= -\epsilon_0 \cdot \frac{1}{r^2 \cancel{\sin\theta}} \frac{\partial}{\partial r}\left(r^2 \cancel{\sin\theta} \frac{\partial V}{\partial r}\right)$$

$$= -\epsilon_0 \frac{1}{r^2} \frac{\partial}{\partial r}\left[r^2 \frac{\partial}{\partial r}\left(-\frac{6r^5}{\epsilon_0}\right)\right]$$

$$= +\frac{\cancel{\epsilon_0}\,\partial}{r^2 \partial r}\left[r^2 \frac{30}{\cancel{\epsilon_0}} r^4\right]$$

$$= \frac{1}{r^2} \times 30 \times 6 r^5$$

$$= 180 r^3$$

Date

$$Q = \iiint_v \rho \, dv$$

145

$$= 180 \iiint r^3 \cdot r^2 \sin\theta \cdot dr \cdot d\theta \cdot d\phi$$

$$= 180 \underbrace{\int_0^1 r^5 \cdot dr}_{=1/6} \underbrace{\int_0^{\pi} \sin\theta \cdot d\theta}_{=2} \underbrace{\int_0^{2\pi} d\phi}_{=2\pi}$$

$$= 180 \times \frac{1}{6} \times 4\pi$$

$$= 120\pi \text{ Ans}$$

Q: 7 $\vec{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$

$\nabla \times \vec{A} = 0$ ---- irrotational vector

$\underbrace{\nabla \cdot \vec{A} \neq 0}$ ---- not solenoidal

Divergence less

Q: 8 $W_e = \dfrac{Q_1 Q_2}{4\pi\epsilon r}$

$W_e \propto \dfrac{1}{r}$

$W_{e_2} = \dfrac{1}{2} W_{e_1}$

Q: 11 Line charge : $(\underbrace{y=3}; \underbrace{z=5})$

$\vec{E}(0, \underline{6, 1}) =$ —

$\vec{E}(5, \underbrace{6, 1}) =$? — remains same

Date

Q:17 $\vec{E} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$ $\qquad V = -\int_{y}^{x} \vec{E}\cdot d\vec{l}$

$$d\vec{l} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$$

$$\vec{E}\cdot d\vec{l} = x\,dx + y\,dy + z\,dz$$

$$V = -\left[\int_1^2 x\,dx + \int_2^0 y\,dy + \int_3^0 z\,dz\right]$$

$V = +5$ Volt

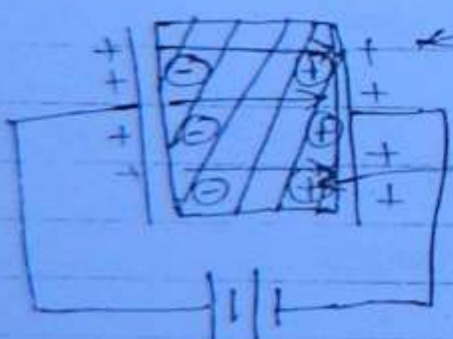
Q:18 $V = 3x^2y - yz$

$V(1,0,-1) = 0$

$$\vec{E} = -\nabla V = -\left[\frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z\right]$$

$\neq 0$

19 Polarization in dielectric materials



← free charge; ρ_s C/m² -- present on conductor

Bound charges; ρ_{sp} — surface charge density due to polarization

$D = \epsilon_0 E$ -- for free space

$D = \epsilon_0 \epsilon_r E$ -- for dielectric

$D = \epsilon_0 E + P$ ---- for dielect.

Date

$D = \epsilon_0 E + P$ (147)

$P = D - \epsilon_0 E = D - \epsilon_0 \frac{D}{\epsilon} = D - \epsilon_0 \frac{D}{\epsilon_0 \epsilon_r}$

$\boxed{P = D\left(1 - \frac{1}{\epsilon_r}\right)} = 2\left(1 - \frac{1}{5}\right) = 1.6 \; C/m^2$

Imp. ① The polarization in the dielet. matrials is present whenever it is subjective to some externally applied elet. field.

② The charges are induced with in the dielet. due to dipole information so that net charge induced in the dielectric slub is zero.

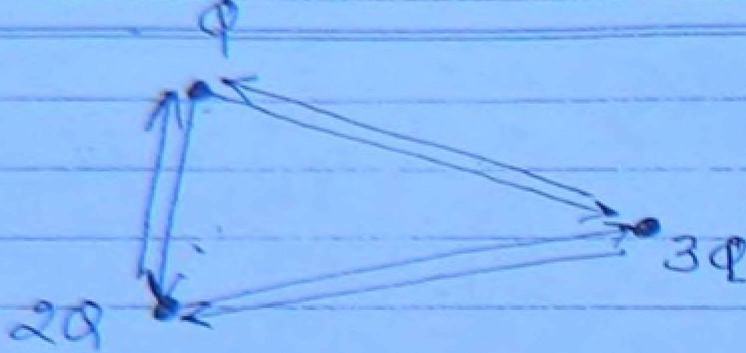
③ Polarization represents total dipole movement information per unit volume of the dielet.

④ Due to induced charges, overall field Distribution in the dielel. is modified.

⑤ it is a temporary phenemonan - so long as externally applied elet. field is present the diet will remain in the polarized stage.

as soon as this eleft field with drawn the dielel. returns to its unpolarized stage & the induced charges are no longer present.

(21)



148

$\vec{F_{13}} + \vec{F_{23}} = 3\vec{F}$ —— (1)

$\vec{f_{12}} + \vec{f_{32}} = 2\vec{F}$ —— (2)

$\vec{f_{21}} + \vec{f_{31}} = ?$

$-\vec{f_{31}} - \cancel{\vec{f_{32}}} = 3\vec{F}$

$-\vec{f_{21}} + \cancel{\vec{f_{32}}} = 2\vec{F}$

$\vec{f_{31}} + \vec{f_{21}} = -5F$ Ans.

Q: 22

$q\vec{E} = qV \times \vec{B}$

$\boxed{\vec{E} = \vec{v} \times \vec{B}}$

Smith chart

① cart. coord system :
(x, y, z)

149

$$d\vec{l} = \begin{cases} \pm\, dx\, \hat{a}_x \\ \pm\, dy\, \hat{a}_y \\ \pm\, dz\, \hat{a}_z \end{cases}$$

$$d\vec{S} = \begin{cases} \pm\, dx\, dy \,.\, \hat{a}_z \\ \pm\, dy\, dz \,.\, \hat{a}_x \\ \pm\, dx\, dz \,.\, \hat{a}_y \end{cases}$$

$$d\vec{V} = dx \cdot dy \cdot dz$$

$$\begin{cases} -\infty < x < +\infty \\ -\infty < y < +\infty \\ -\infty < z < +\infty \end{cases}$$

② cyl. coord. system: (r, ϕ, z)

$$d\vec{l} = \begin{cases} \pm\, dr\, \hat{a}_r \\ \pm\, r \cdot d\phi\, \hat{a}_\phi \\ \pm\, dz \end{cases}$$

$$d\vec{S} = \begin{cases} \pm\, r\, dr\, d\phi \cdot \hat{a}_z \\ \pm\, r\, d\phi\, dz\, \hat{a}_r \\ \pm\, dr\, dz \cdot \hat{a}_\phi \end{cases}$$

$dV = r \cdot dr \cdot d\phi \cdot dz$

$$\begin{cases} 0 \le r < \infty \\ 0 \le \phi < 2\pi \\ -\infty < z < +\infty \end{cases}$$

(156)

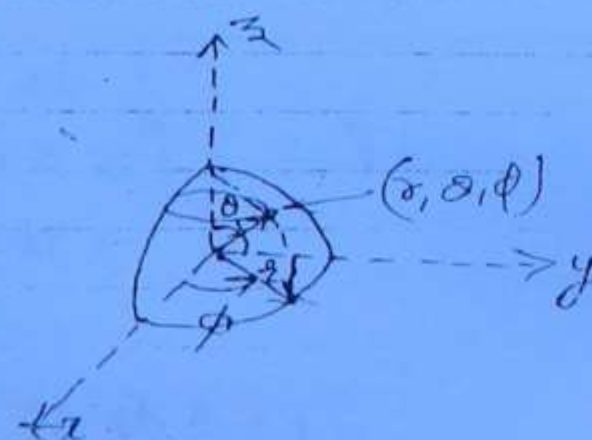
(3) Sph. coord. system :- (r, θ, ϕ)

$$d\vec{l} = \begin{cases} \pm dr \cdot \hat{a}_r \\ \pm r\, d\theta\, \hat{a}_\theta \\ \pm r \sin\theta\, d\phi\, \hat{a}_\phi \end{cases}$$

$$d\vec{S} = \begin{cases} \pm r\, dr\, d\theta \cdot \hat{a}_\phi \\ \pm r^2 \sin\theta \cdot d\theta\, d\phi \cdot \hat{a}_r \\ \pm r \sin\theta \cdot dr\, d\phi\, \hat{a}_\theta \end{cases}$$

$dV = r^2 \sin\theta \cdot dr \cdot d\theta \cdot d\phi$

$$\begin{cases} 0 \le r < \infty \\ 0 \le \theta \le \pi \\ 0 \le \phi \le 2\pi \end{cases}$$



General relations

Date

(u, v, w)

1. $\nabla V = \sum \frac{1}{h_1} \frac{\partial V}{\partial u} \hat{a}_u$ (151)

2. $\nabla \cdot \vec{A} = \frac{1}{h_1 h_2 h_3} \sum \frac{\partial}{\partial u} (h_2 h_3 A_u)$

3. $\nabla^2 V = \frac{1}{h_1 h_2 h_3} \sum \frac{\partial}{\partial u} \left(\frac{h_2 h_3}{h_1} \frac{\partial V}{\partial u} \right)$

4. $\nabla \times \vec{A} = \frac{1}{h_1 h_2 h_3} \begin{vmatrix} h_1 \hat{a}_u & h_2 \hat{a}_v & h_3 \hat{a}_w \\ \frac{\partial}{\partial u} & \frac{\partial}{\partial v} & \frac{\partial}{\partial w} \\ h_1 A_u & h_2 A_v & h_3 A_w \end{vmatrix}$

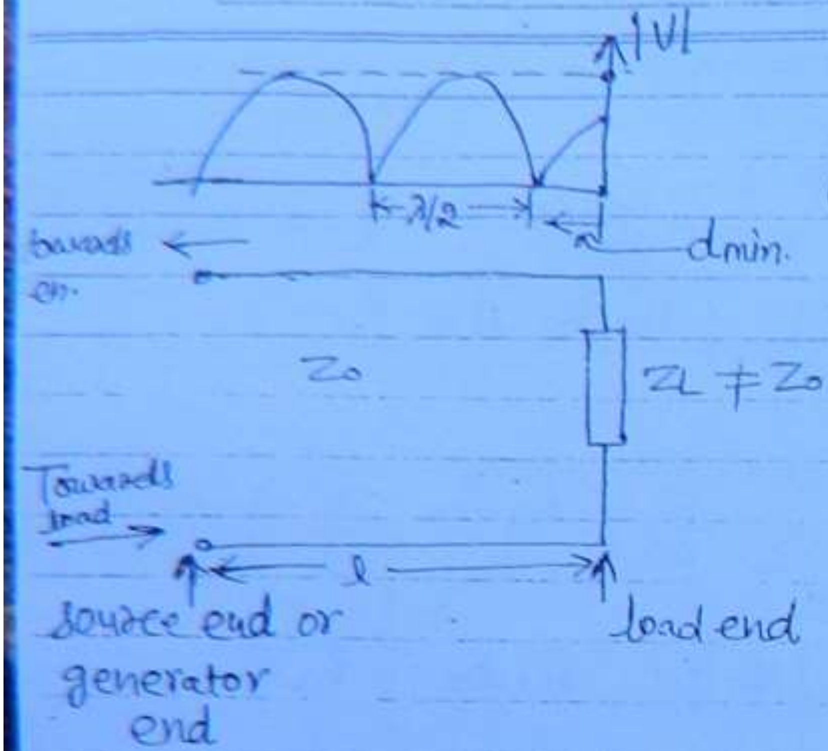
$\vec{A} = A_u \hat{a}_u + A_v \hat{a}_v + A_w \hat{a}_w$

	u	v	w	h_1	h_2	h_3
cart.	x	y	z	1	1	1
cyl.	r	ϕ	z	1	r	1
sph.	r	θ	ϕ	1	r	$r\sin\theta$

Smith chart :-

---- T.L Calculate

Date



Zmin
Vmin
Imax

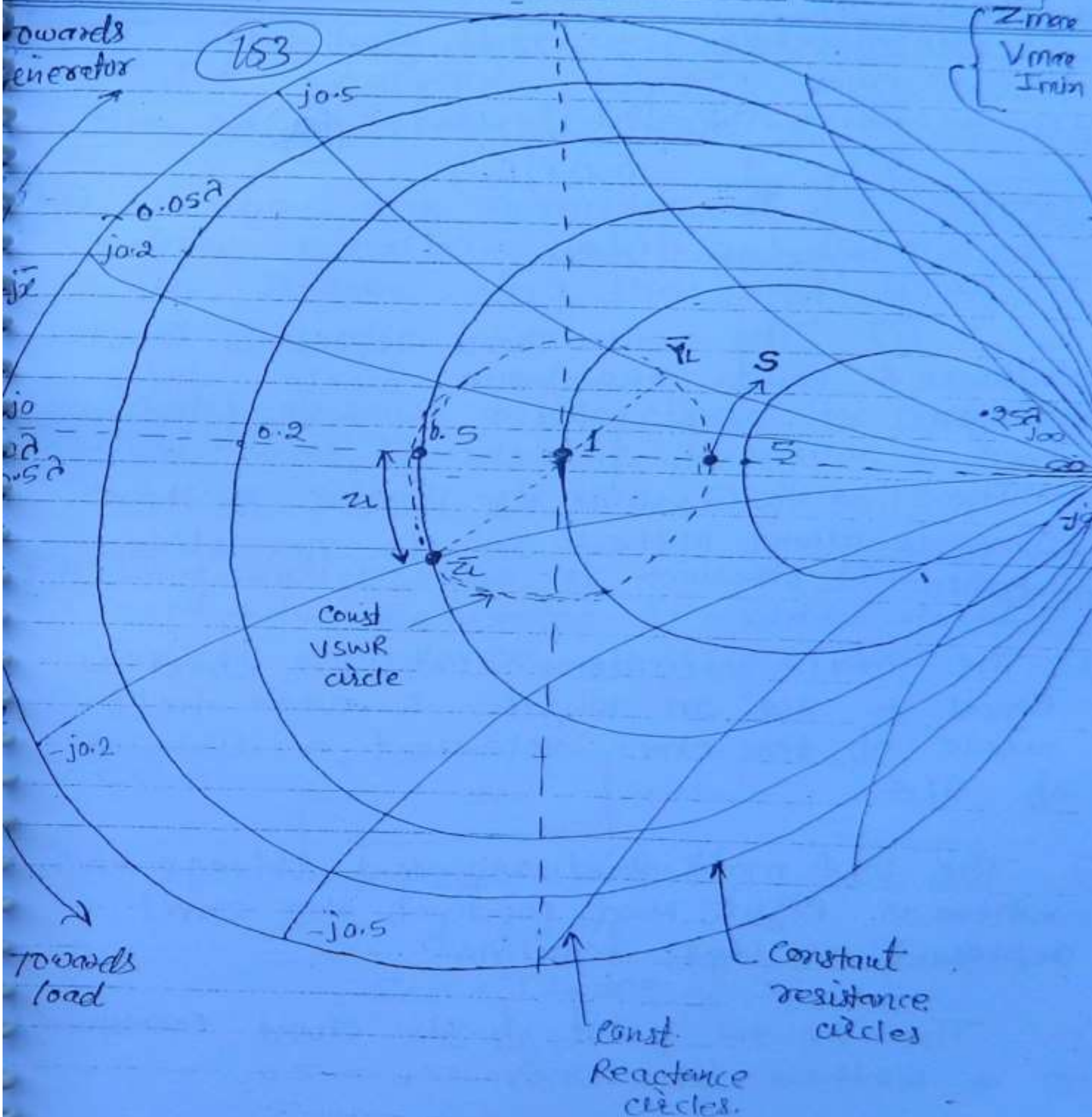
(152)

1. lossless line
2. normalized impedance

$$\left(\frac{Z}{Z_0}\right) \equiv \bar{Z} = \bar{r} \pm j\,\bar{x}$$

VSWR
$1 \to \infty$

Date



Imp. points

Z_L = Given ; Z_0 -- given

$\bar{Z}_L = 0.5 - j0.2$ (154)

$\bar{Y}_L = \frac{1}{\bar{Z}_L}$ = (red) + j(--)

S ; dmin ; dmax.

(1) The smith chart represents const. Resistance & const. reactance circle. which are orthogonal at each point.

(2) The line is assume as lossless & the imp. is alway plotted in its normalized form.

(3) The total circumference of the chart is equal to $\lambda/2$ in length & each half circle of the chart represent a distance of $\lambda/4$.

(4) The left most point represent Voltage minima whereas Right most point of the chart represent voltage maxima.

(5) The centre point of the chant corresponce to a Mathed line when $Z_L = Z_0$.

(6) The total Distance b/w the centre of the chart to the Right most of point represent total range of VSWR from $1 \to \infty$

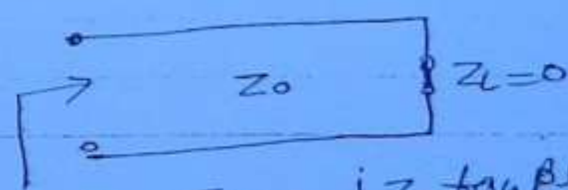
(7) upper half the circle corraspondce to +ive Reactance wheras lower half of the circle represents negative reactance

(8) To find the normilazed Admittance from normilozed imp. a distance of $\lambda/4$ is move along the const VSWR circle.

(9) Going closewise in the chart the impedance are moves towards the generator & Inductive imp.s are added to the initial impedance

(10) going anticlosewise or towards the load capacitive Reactane is added to the initial impedance along the line.

Ex:

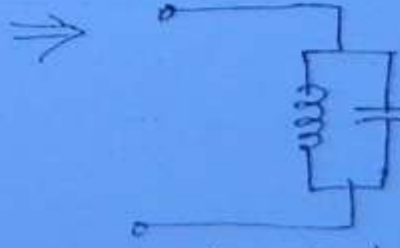


$$Z_{in} = Z_{sc} = j Z_0 \tan \beta l$$

if $l = \lambda/4$

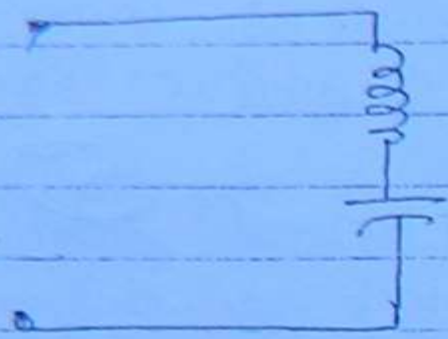
$$Z_{sc} = j Z_0 \tan \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4}$$

$$= j\infty$$



parallel LC Ressonance

$$Z_{oc} = -j Z_0 \cot \beta l \Big|_{l = \lambda/4} = 0$$



--- Series LC Resonance circuit

Conculsion :

(1) for $\lambda/4$ section of the line a short circuit line represent a parallel LC resonant ckt. Where as an open circuit line represent a series LC resonant circuit.

